Brushless DC Motor Drive
Why Brushless DC motor?

• The maximum speed and torque of the Brushed DC Motor is limited by the carbon brush-commutator.

• However, the control of speed and torque of a DC motor is the simplest, because the torque characteristic is linear with $i_a$ and $i_f$.

• The brushless structure has all the linear characteristics of the brushed DC motor but without the complexity and maintenance requirement of the brush-commutator.
Construction of brushed and brushless DC machines

Brushed DC Motor
- Stator - Field
- Rotor - Armature
- 4-poles

Brushless DC Motor
- Stator - Armature
- Rotor - Field
- 2-poles
- 4-poles
Permanent magnet Brushed and Brushless DC motor

The brush-commutator is replaced by a 3-phase *electronic commutator*, consisting of three Hall sensors for rotor position and a 3-phase inverter.
Permanent magnets for BLDC motor

Neodymium Iron Boron (NdFeB) and Samarium Cobalt

NdFeB: $B_r = 0.8 - 1.4 \, T; \quad H_c > 1 \, MA/m; \quad \mu_{recoil} \approx 1; \quad T_{Curie} = 120 \, ^\circ C$
Three phase stator
\[
\theta = \int_0^t \omega_m \, dt + \theta_o
\]
The Back EMF waveform

\[ e = -N_s \frac{d\varphi_f}{dt} = Blv = Blr\omega_m \]
Phase & line-line voltages
Machine back emf

For the two-pole \((p = 1)\) machine, \( e = -N_s \frac{d\phi}{dt} = Blv = Blr\omega_m \)

\[ e_{\text{max}} = 2N_s Blv = 2N_s lrB\omega_m \text{ V} \]

where \( l \) is the active length of the conductors in meters
\( r \) is the radius of the stator in meters
\( \omega_m \) is the rotational speed of the rotor in rad/sec.
\( B \) is the air-gap field in Tesla
\( N_s \) is the number of conductors per pole

For a star-connected machine with \( p \) pole-pairs (i.e., \( 2p \) poles),

\[ e_{\text{max}} = 2N_s plrB\omega_m = k_e' \varphi_f \omega_m \text{ V/phase} \]

\[ e_{ab\text{max}} = \text{max}(e_a - e_b) = 4N_s plrB\omega_m = k_E' \varphi_f \omega_m \text{ V}_{\text{line-line}} \]
\[ T = \frac{e_a i_a + e_b i_b + e_c i_c}{\omega_m} \]
Inverter switching table for \( \omega > 0 \), or CCW motion

<table>
<thead>
<tr>
<th>( \delta )</th>
<th>Switches ON</th>
<th>Current flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>-30° - 30°</td>
<td>T5, T6</td>
<td>( C^+ ) and ( B^- )</td>
</tr>
<tr>
<td>30° - 90°</td>
<td>T6, T1</td>
<td>( B^- ) and ( A^+ )</td>
</tr>
<tr>
<td>90° - 150°</td>
<td>T1, T2</td>
<td>( A^+ ) and ( C^- )</td>
</tr>
<tr>
<td>150° - 210°</td>
<td>T2, T3</td>
<td>( C^- ) and ( B^+ )</td>
</tr>
<tr>
<td>210° - 270°</td>
<td>T3, T4</td>
<td>( B^+ ) and ( A^- )</td>
</tr>
<tr>
<td>270° - 330°</td>
<td>T4, T5</td>
<td>( A^- ) and ( C^+ )</td>
</tr>
</tbody>
</table>
## Inverter switching table

Inverter switching table for $\omega < 0$, or CW motion

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>Switches ON</th>
<th>Current flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>-30º - 30º</td>
<td>T5, T6</td>
<td>C+ and B–</td>
</tr>
<tr>
<td>30º - 90º</td>
<td>T6, T1</td>
<td>B– and A+</td>
</tr>
<tr>
<td>90º - 150º</td>
<td>T1, T2</td>
<td>A+ and C–</td>
</tr>
<tr>
<td>150º - 210º</td>
<td>T2, T3</td>
<td>C– and B+</td>
</tr>
<tr>
<td>210º - 270º</td>
<td>T3, T4</td>
<td>B+ and A–</td>
</tr>
<tr>
<td>270º - 330º</td>
<td>T4, T5</td>
<td>A– and C+</td>
</tr>
</tbody>
</table>
\[ T_1 = SA \& \bar{SB} \]
\[ T_3 = SB \& \bar{SC} \]
\[ T_5 = SC \& \bar{SA} \]
\[ T_4 = \bar{SA} \& SB \]
\[ T_6 = \bar{SB} \& SC \]
\[ T_2 = \bar{SC} \& SA \]
The developed torque

For a 2-pole machine \((p = 1)\)

\[
T = 2N_s r l B I \quad \text{Nm/phase}
\]

\[
= 4N_s r l B I \quad \text{Nm}
\]

For a machine with \(p\) pole-pairs (i.e., \(2p\) poles)

\[
T = 4N_s p r l B I = k_T \varphi_f I \quad \text{Nm}
\]

Note that the back emf and torque equations of this AC machine are exactly the same as for the brushed DC machine. There are no brushes or commutator. Hence the name *Brushless DC Machine*. 
Ideally, the DC-link currents (+ve and –ve) should remain constant, at all times. This can be achieved by one DC current regulator, or three separate phase current regulators.
DC link voltage versus speed

Assuming quasi-square phase currents in each winding, via DC-link or phase current controls

\[ V_d I_d = 3VI \cos \phi \]
Fourier analysis of phase current
Phase and DC-link current waveforms
Phase current harmonics

\[ a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} i \cos(n\omega t) \, d\omega t \]

\[ = \frac{1}{\pi} \int_{-\pi/3}^{\pi/3} I_d \cos(n\omega t) \, d\omega t + \frac{1}{\pi} \int_{-\pi}^{-2\pi/3} -I_d \cos(n\omega t) \, d\omega t + \frac{1}{\pi} \int_{2\pi/3}^{\pi} -I_d \cos(n\omega t) \, d\omega t \]

\[ = \frac{2I_d}{n\pi} \left[ \sin \left(\frac{n\pi}{3}\right) + \sin \left(\frac{2n\pi}{3}\right) \right] \]

\[ = \frac{4I_d}{n\pi} \cos \left(\frac{n\pi}{6}\right) \quad (n=1,3,5) \]

\[ I_1 = a_1 / \sqrt{2} = \frac{4I_d}{\sqrt{2\pi}} \cos \left(\frac{\pi}{6}\right) = \frac{\sqrt{6}I_d}{\pi} \]

\[ \therefore I_d = \frac{\pi}{\sqrt{6}} I \quad \text{where } I \text{ is the RMS value of the fundamental phase current. Note, } I \text{ is also the peak value.} \]
Per-phase Phasor diagram

(a) Circuit diagram with\n- Current $I$
- Resistance $R$
- Inductive reactance $j\omega L_s$
- Voltage $V$
- Generator EMF $E$

(b) Per-phase phasor diagram for\n- Voltage $V$
- Current $I$
- Inductive reactance $I\omega L_s$
- Load angle $\phi$
- Voltage drop $E$

(c) Per-phase phasor diagram for\n- Voltage $V$
- Current $I$
- Inductive reactance $I\omega L_s$
- Load angle $\gamma$
- Voltage drop $E$
Relationship between $V_d$ and $\omega_m$

\[ V \cos \phi = E_f \cos \gamma + IR \; ; \; V_d I_d = 3VI \cos \phi = 3IE \cos \gamma + 3I^2R \]

Noting that \( I_d = \frac{\pi}{\sqrt{6}} I \) and cancelling \( I \),

\[ 0.427V_d = E \cos \gamma + IR \]

When \( \gamma = 0^\circ \), \( 0.427V_d = E + IR = V'_d \) Thus, for small \( IR \), \( \omega_m \propto V_d \)

$V'_d$ is the per phase RMS voltage supplied to the motor by the inverter.