Section 5: Induction Motor Drive

1. Brief review of IM theory.

2. IM drive characteristics with:
   - Variable input voltage
   - Variable rotor resistance
   - Variable rotor power
   - Variable voltage and variable frequency, VVVF drive (VSI \( V/f \) inverter drive)
   - Variable current and variable frequency, VCVF drive (CSI \( I/f \) inverter drive)
Introduction

- Induction machines are the most widely used in industry because of its ruggedness, low maintenance. The IM is also cheaper than most other electric motors. Covers applications from a few Watts to mega Watts.

- Traditionally, used as-
  - constant speed drive (without an inverter)
  - variable speed $V/f$ (Volts/Hertz) drive (with slow dynamics)

- Recent developments in control techniques and power electronics has made it possible for the Induction Motor (IM) to be used in applications requiring fast dynamic response and decoupled control of torque and flux, like the brushed DC motor.
Physical Structure of the Induction Motor
Stator and Rotor

3-phase Sinusoidally Distributed Stator winding
Cage rotor
Wound rotor
Working Principle

- 3-phase balanced currents of a certain frequency in three-phase stator windings leads to

Rotating Magnetic Field

- Speed of the rotating magnetic field is the synchronous speed,

\[ N_{\text{syn}} = \frac{f_1}{p} \text{ rev/sec}; \quad \omega_{\text{syn}} = \frac{2\pi f_1}{p} = \frac{\omega_1}{p} \text{ mech rad/sec} \]

- Because of this rotating field, voltage is induced in the rotor windings (or aluminum bars). The consequent 3-phase current flow in the rotor establishes a rotor field.

- Interaction between rotor and stator fields will produce the necessary torque to rotate the rotor and load with speed \( n_{\text{rot}} \).

\[ N_{\text{rot}} = \frac{f_2}{p} \text{ rev/sec}; \quad \omega_{\text{rot}} = \frac{2\pi f_2}{p} = \frac{\omega_2}{p} \text{ mech rad/sec} \]
The revolving field of the rotor rotates at slip frequency with respect to rotor. This field also rotates at synchronous speed, $N_{syn}$, with respect to the stator because of rotation of the rotor.
Slip and slip frequency

\[ s = \frac{N_{syn} - N_{rot}}{N_{syn}} = \frac{\omega_{syn} - \omega_{rot}}{\omega_{syn}} \]

Slip frequency, \( f_r = sf_1 = f_1 - f_2 \)

\( s = 1 \) when the rotor is at standstill.
\( s = 0 \) when the motor runs at synchronous speed. Possible ???
\( s \approx 0.025 - 0.07 \), normally. The slip frequency, \( sf_1 \), is the frequency of the voltage and current induced in the rotor.

Rotor induced voltage/phase:

\[ E_2 = 4.44N_r \phi_r f_r = 4.44N_r \phi_r sf_1 \]

Rotor voltages and currents are at the slip frequency \( sf_1 \).
Representation of the Rotor Circuit

At slip frequency

\[ R'_2 = a^2 R_2; \quad X'_2 = a^2 X_{2s\text{standstill}} \]

\[ E_1 = aE_2 = E'_2 \]

\[ I'_2 = \frac{I_2}{a} \]

At stator frequency

\[ I'_2 = \frac{I_2}{a} \]

\[ X'_2 \]

\[ R'_2 \]

Mechanical load

Rotor circuit Referred to stator

\[ E_1 = aE_2 = E'_2 \]

\[ \frac{R'_2 (1-s)}{s} \]
The approximate equivalent circuits

\[ I_2' = \frac{I_2}{a} \]

\[ E_1 = aE_2 = E_2' \]

Total Rotor Power: \[ P_2 = 3I_2'^2 \frac{R_2'}{s} = \frac{3sR_2'E_1^2}{R_2'^2 + \left(s \omega_1 L_2'\right)^2} \ W \]

Developed Output Power: \[ P_o = P_2 - 3I_2'^2 R_2' = 3I_2'^2 \frac{R_2'(1-s)}{s} = (1-s) P_2 \ W \]
Developed power and torque

Slip Power: \( P_{sl} = P_2 - P_o = sP_2 = 3I_2'^2R_2' \) W

Developed torque = Developed output power/mech speed in rad/sec:

\[
T_{dev} = \frac{P_o}{2\pi N_{rot}} = \frac{3I_2'^2R_2'(1-s)/s}{2\pi N_{rot}} \quad \text{Nm}
\]

\[
= \frac{3I_2'^2R_2'(1-s)/s}{2\pi N_{syn}(1-s)} \quad \text{Nm}
\]

\[
= \frac{3pI_2'^2R_2'}{2\pi f_1 s} = \frac{3pI_2'^2R_2'}{2\pi (f_1 - f_2)} \quad \text{Nm}
\]

\[
= \frac{\text{Rotor Power} (P_2)}{\text{Syn Speed}} = \frac{\text{Slip Power} (P_{sl})}{\text{Slip Speed}} \quad \text{Nm}
\]
Note that for \( X_m \gg (R_I \text{ and } X_I) \); \( R_{Th} \approx R_I \); \( X_{Th} \approx X_I \); and \( V_{Th} \approx V_I \). Note also that \( R_c \) is not included in this representation. The rotational loss that \( R_c \) represents is included in the no-load slip \( s_{nl} \).
Rotor current and torque

\[ I_2' = \frac{V_{Th}}{\sqrt{\left( R_{Th} + \frac{R_2'}{s} \right)^2 + \left( X_{Th} + X_2' \right)^2}} \quad \text{A} \]

\[ T_{dev} = \frac{3p}{\omega_1} \times \frac{V_{Th}^2}{\left( R_{Th} + \frac{R_2'}{s} \right)^2 + \left( X_{Th} + X_2' \right)^2} \times \frac{R_2'}{s} \quad \text{Nm} \]
Torque-speed characteristics of an IM with variable voltage

\[ P = T \omega \]
Braking of an IM drive

Two ways:

- By plugging.
- By adjusting input frequency below shaft frequency $f_2$. This requires an inverter.

Note: Operation with high slip causes high power loss; may lead to high rotor temperature as a consequence.
$T_{\text{max}}$ and slip $s_{mT}$ for $T_{\text{max}}$

For small slip, \[ T_{\text{dev}} \approx \frac{3p}{\omega_1} \times \frac{V_{Th}^2}{R_2} \times s \quad \text{Nm} \]

For maximum torque, \[ \frac{R_2'}{s_{mT}} = \sqrt{\left[ R_{Th}^2 + \left( X_{Th} + X_2' \right)^2 \right]} \]

Slip for maximum torque, \[ s_{mT} = \frac{R_2'}{\sqrt{R_{Th}^2 + \left( X_{Th} + X_2' \right)^2}} \]

Maximum torque, \[ T_{\text{max}} = \frac{3p}{2\omega_1} \times \frac{V_{Th}^2}{R_{Th} + \sqrt{R_{Th}^2 + \left( X_{Th} + X_2' \right)^2}} \quad \text{Nm} \]

Note that $T_{\text{max}}$ is independent of $R_2'$

ELEC4613 – Electric Drive Systems
IM torque characteristic with $R_2'$
Variable-speed Induction Motor drives
IM drive with variable supply voltage

Variable AC voltage at the mains supply frequency can be obtained from tap-changing transformer, from back-back phase-controlled thyristor converter or from an inverter.

\[ V_I = \frac{V_{\text{max}} l-n}{\sqrt{2}} \sqrt{\frac{1}{\pi} \left( \pi - \alpha + \frac{\sin 2\alpha}{2} \right)} \]
Variable voltage operation at the utility supply (base) frequency offers very limited speed range. Pump type loads are suitable; however, high-slip and very lossy operation is inevitable with reduced supply voltage.
Ex 1: Voltage control for a fan or compressor loads

Fan or compressor type load:

Shaft Power: \[ P_o = T \omega_m = K(1 - s)^2(1 - s) = K(1 - s)^3 \]

Rotor Developed Power: \[ P_2 = \frac{P_o}{1 - s} = K(1 - s)^2 \]

Slip Power: \[ P_{sl} = I_2' R_2' = sP_2 = Ks(1 - s)^2 \]

\[ \therefore I_2' = (1 - s) \sqrt{\frac{Ks}{R_2'}} \]

For maximum \( P_{sl} \): \( s = 0.333 \)
Ex 2: constant load

For constant torque type load:

\[ P_o = T\omega_m = K(1 - s) \]

\[ P_2 = \frac{P_o}{1 - s} = K \]

\[ P_{sl} = I_2'^2 R_2' = sP_2 = Ks \quad \therefore \quad I_2' = \sqrt{\frac{Ks}{R_2'}} \]

Examples 1 & 2 show that, the rotor current or rotor power loss increases less slowly with slip (or load) for a fan or compressor type load than with a constant-torque type load.
WRIM drive with variable rotor power

AC Mains

Wound Rotor IM

Slip Rings

Variable Resistor Bank

Rotor terminals

Tank 2

Valve 1

Valve 2

Tank 1

Pump

ELEC4613 – Electric Drive Systems
Figure 5.2.4. $T$-$\omega$ characteristic with variable rotor resistance.

Speed and torque can be reduced by increasing $R_2'$, however, lower speed operation is with increased slip, implying increased rotor power loss.
IM drive with variable rotor power (slip power control – static Scherbius scheme)
**T - \( I_d \) characteristic with control of DC link current**

Slip Power, \( P_{sl} = P_2 - P_o = \frac{3I_2'^2R_2'}{s} - \frac{3I_2'^2R_2'(1-s)}{s} = 3I_2'^2R_2' = sP_2 \) W

Output Power, \( P_o = (1-s)P_2 \) W

The diode rectifier output DC voltage, \( V_d = \frac{3sE_{2max,l-l}}{\pi} \)

Note that \( E_{2max,l-l} \) is peak-line-line rotor voltage at standstill (for \( s = 1 \)).

Thus, \( V_d = \frac{3\sqrt{3}\sqrt{2} \times sE_2}{\pi} \) where \( E_2 \) is the rotor RMS voltage/phase.

If the stator voltage drop is negligible, \( E_2 \approx V_1/a, \) \( V_d = \frac{3\sqrt{3}\sqrt{2} \times sV_1}{\pi a} \)

Also, if the rotor resistance \( R_2 \) is small i.e., the DC power is large compared to the power loss in rotor resistance \( R_2 \),

\[
P_{sl} = sP_2 = V_d I_d = \frac{3\sqrt{6} \times sV_1}{\pi a} I_d
\]
Thus, \[ P_2 = \frac{3\sqrt{6} \times V_1}{\pi a} I_d \]

The developed torque, \[ T = \frac{p}{\omega_s} \times \frac{3\sqrt{6} \times V_1}{\pi a} I_d \] Nm

\[ T \approx K I_d \]

Thus, the developed torque can be controlled via the DC link current \( I_d \), when the slip power is small compared to the total rotor (or air-gap) power \( P_2 \).
Speed control with slip power recovery

\[ P_2 = P_o + P_{sl} + P_{ret} \quad P_o = (1 - s)P_2 \]

\[ s = \frac{P_{sl} + P_{ret}}{P_2} \]

By neglecting the voltage drop across stator impedance, \( E_2 \approx \frac{V_1}{a} \)

The DC output voltage of rectifier, \( V_d = \frac{3sE_{2\text{max},l-1}}{\pi} = \frac{3\sqrt{6}sV_1}{\pi a} \) \hspace{1cm} (1)

\[ V_{di} = -\frac{3\sqrt{6}V_2}{\pi} \cos \alpha = -\frac{3\sqrt{6}V_1}{\pi n} \cos \alpha \quad \text{for} \; \alpha > 90^\circ \] \hspace{1cm} (2)

From (1) and (2), and equating \( V_d = V_{di} \) because the DC link inductors supports no DC voltage,

\[ \therefore s = -\frac{a}{n} \cos \alpha ; \quad \text{Normally,} \; n \approx a. \; \text{Why?} \]
Speed control with slip power recovery

Figure 5.2.8.

![Graph showing speed control with slip power recovery with different torque values for different angles.](image)
Step Response of dc link current control loop

Rotor current during step change

Stator current during step change
IM drive with 3-phase VSI $\text{VVVF}$ inverter

\[ V_{An,1} = m \cdot \frac{V_d}{2\sqrt{2}} = 0.354mV_d \]

where \( m \) is the depth of modulation
Performance with VVVF \((V/f)\) supply

We assume that the AC supply voltage to the motor is sinusoidal, but of arbitrarily variable amplitude (RMS value \(V_1\)) and frequency \(f_1\).

\[
f_1 = \lambda f_0 \quad 0 \leq \lambda \leq 1
\]

for operation from zero to base speed. \(\lambda\) is higher than 1 for operation above base speed.

---

Figure 5.2.11

\[
\begin{align*}
I_1 & \quad R_1 & \quad \lambda X_1 & \quad I'_1 \\
E_1 & \quad \lambda X_m & \quad I_m & \quad R'_2 \left( \frac{1 - s}{s} \right) \\
V_1 & \quad \lambda X'_2 & \quad I'_2 & \quad R_2
\end{align*}
\]
VVVF (or $V/f$) drive with constant air gap flux

\[ V_1 = R_1 I_1 + j \omega_1 L_1 I_1 + E_1 = R_1 I_1 + j \omega_1 L_1 I_1 + K \hat{\phi}_{ag} f_1 \]

For operation near base speed, the stator voltage drop: \( R_1 I_1 + j \omega_1 L_1 I_1 \) can be neglected, compared to \( V_1 \).

\[ V_1 \approx K \hat{\phi}_{ag} f_1 \quad K \hat{\phi}_{ag} = \frac{V_1}{f_1} \]

Thus, for operation near base speed, constant $V/f$ supply implies operation with constant air-gap flux.
$T$-ω characteristic with constant $V/f$ drive

With negligible stator impedance drop, 

$$I'_2 = \frac{E_1}{R'_2 + j \omega_1 L'_2} \approx \frac{V_1}{R'_2 + j \omega_1 L'_2} = \frac{sV_1}{R'_2 + j 2\pi s f_1 L'_2}$$

$$|I'_2| = \frac{s f_1 V_o}{f_o} \sqrt{R'_2 + \left(s f_1 2\pi L'_2\right)^2} = \frac{s f_1 K \hat{\phi}_{ag}}{\sqrt{R'_2 + \left(s f_1 2\pi L'_2\right)^2}}$$

$$T_{dev} = \frac{3p s R'_2 \left( K \hat{\phi}_{ag} f_1 \right)^2}{\omega_1 R'_2 + \left(s \omega_1 L'_2\right)^2} = \frac{3p s f_1 R'_2 \left( K \hat{\phi}_{ag} \right)^2}{2\pi R'_2 + \left(2\pi s f_1 L'_2\right)^2} \text{Nm}$$

Thus, $I'_2$ and $T_{dev}$ values remain the same for a given $sf_1$, regardless of $f_1$. 

ELEC4613 – Electric Drive Systems
IM drive with constant $V/f$ ratio

$T_{dev}$

Slip freq, $sf_1$

Speed, Rev/min

Torque, Nm

Rotor current $I'_2$, A

ELEC4613 – Electric Drive Systems
Starting with maximum torque, $V/f$ drive

For maximum developed power and torque in the rotor circuit

$$\frac{R'_2}{s} = \lambda X'_2 \quad \frac{R'_2 f_1}{f_1 - f_2} = \frac{f_1}{f_o} X'_2$$

When $T_{max}$ is developed, $\Rightarrow$

$$sf_1 = f_1 - f_2 = \frac{f_o R'_2}{X'_2}$$

Note: maximum torque occurs at the same slip frequency for all $f_1$.

For maximum torque to occur at zero speed,

$$f_1 = \frac{f_o R'_2}{X'_2} = \frac{R'_2}{2\pi L'_2}$$

From 5.3.26 and 5.2.27,

$$T_{max} = \frac{3p \left( K\phi_{ag} \right)^2 f_o}{4\pi X'_2} \text{ Nm}$$
IM drive with constant $V/f$ ratio

Rated $V_1$

$K\hat{\phi}_{gap}$

$f_0$ $f_1$
Constant max torque and power characteristics

Figure 5.2.14. \( T - \omega \) characteristics under VVVF drive with \( f_1 \) below and above \( f_o \).

ELEC4613 – Electric Drive Systems
$T-\omega$ characteristics with VSI $V/f$ drive

$\omega$, rad/sec

$T-\omega$ characteristic with rated $V_o$ and base $f_o$

Sequence: $a-b-c$

$Q_2$

$Q_1$

Sequence: $a-c-b$

$Q_3$

$Q_4$

$T$, Nm

$0$

$T_{rated}$

$T_{max}$

ELEC4613 – Electric Drive Systems
**V/f drive at low speed**

\[ V_1 = R_1 I_1 + j\omega_1 L_1 I_1 + E_1 = R_1 I_1 + j\omega_1 L_1 I_1 + K\hat{\phi}_{ag} f_1 \]

At low speed, the stator impedance drop \( R_1 I_1 + j\omega_1 L_1 I_1 \) may not remain negligible compared to \( V_1 \) or \( E_1 \). It implies reduction of the air-gap flux \( \hat{\phi}_{ag} \), and consequent reduction of \( T_{dev} \)

\[
T_{dev} = \frac{3p}{\omega_1} \frac{sR_2 \left( K\hat{\phi}_{ag} f_1 \right)^2}{R_2^2 + (s\omega_1 L_2)^2}
\]
**V/f drive at low speed**

Figure 5.3.1. Drooping $T-\omega$ characteristic at low speed with VVVF drive
The zero-frequency boost is \( V_{bo} = R_1 \times I_{1rated} \)
VSI $V/f$ drive controller – open loop

Speed reference

\[ \frac{1}{1 + T_f s} \]

$V_1$ Reference

$f_1$ Reference

Open-loop $V/f$ controller
Speed control with an inner slip loop

\[
I_2' = \frac{E_1}{R_2' + j\omega L_2'} \approx \frac{V_1}{sR_2' + j\omega L_2'} = \frac{sK \hat{\phi}_ag}{\sqrt{R_2^2 + (sf_1 2\pi L_2')^2}}
\]

Fig 5.3.5. Closed-loop speed controller with inner slip control
IM drive with current sources (CSI drive)
IM drive with variable $I$-$f$ supply

Figure 5.4.1. Per-phase equivalent circuit with current source input
IM drive with variable \( I-f \) supply

\[ T_{\text{dev}} = \frac{3 p I_2^2}{2 \pi f_1} \frac{R_2'}{s} \]

Torque is inversely proportional to slip frequency

![Electric Drive Circuit Diagram](image-url)
IM drive with variable $I-f$ supply

Maximum rotor power and hence developed torque occurs when

$$\frac{R'_2}{s} = \lambda \left( X_m + X'_2 \right)$$
$$\frac{R'_2 f_1}{f_1 - f_2} = \frac{f_1}{f_o} \left( X_m + X'_2 \right)$$

When $T_{max}$ is developed

$$s f_1 = f_1 - f_2 = \frac{f_o R'_2}{X_m + X'_2}$$

Normally, $X_m >> X'_2$. $S_{mT}$ for CSI drive is much smaller than for VSI.

For starting from standstill with $T_{max}$

$$\lambda = \frac{R'_2}{X_m + X'_2}$$

For maximum torque to occur at start

$$f_1 = \frac{f_o R'_2}{X_m + X'_2} = \frac{R'_2}{2\pi \left( L_m + L'_2 \right)}$$
IM drive with variable $I$-$f$ supply contd.

$$T_{dev} = \frac{3}{2\pi} \frac{p I_2^2}{f_1 - f_2} \frac{R'}{f_1 - f_2}$$

For a given $I_1$, the rotor current $I_2'$ is given by (using current division)

$$I_2' = \frac{j \lambda X_m I_1}{R_2'} + j \lambda \left( X_m + X_2' \right)$$

Using the slip condition for maximum torque,

$$s_{mT} = \frac{R_2'}{\lambda \left( X_m + X_2' \right)}$$

$$T_{\text{max}} = \frac{3}{4\pi} \times \frac{X_m^2 I_1^2}{\left( X_m + X_2' \right) f_o} \text{ Nm}$$
$T-\omega$ characteristic with CSI drive
**I-f** drive with constant air-gap flux

\[
I_m = I_1 \times \left( \frac{R_2'}{s} + j \lambda X_2' \right)
\]

Where \( R_2' = R_2 + j \lambda \left( X_m + X_2' \right) \)

\[
|I_m| = |I_1| \times \sqrt{\left( \frac{R_2'^2 + \left( 2 \pi s f_1 L_2' \right)^2}{R_2'^2 + \left( 2 \pi s f_1 \left( L_m + L_2' \right) \right)^2} \right)}
\]
$I_1$ for constant air-gap flux operation

$I_1$ in reverse sequence

No load $I_1$
$I_1$ at no-load

\[ E_1 = \lambda X_m I_m = K\phi_{ag} f_1 \quad \iff \quad K\phi_{ag} = \frac{E_1}{f_1} \approx \frac{V_{1\text{rated}}}{f_o} \]

\[ K\phi_{ag} = \frac{E_1}{f_1} = \frac{\lambda X_m I_m}{f_1} = \frac{V_{1\text{rated}}}{f_o} = \frac{X_m I_m}{f_o} \]

$X_m$ is the magnetizing reactance at base frequency $f_o$.

\[ I_m = I_{1,\text{no load}} = \frac{V_{1\text{rated}}}{X_m} \]

ELEC4613 – Electric Drive Systems 52
Figure 5.4.5. Variable current, variable frequency inverter drive scheme.
CSI $I/f$ drive for large IM machines
Figure 5.4.7. Motor current waveforms and thyristor switching states for a current source drive.