Solution to Tutorial 5 – Isolated DC-DC Converters & Inverters

1.

The maximum value of \( L_m \) which will ensure discontinuous conduction (i.e. complete demagnetisation) is given by

\[
L_m = \frac{(1 - D)^2}{2 f_s} R \left( \frac{N_1}{N_2} \right)^2
\]

For a higher value of \( L_m \), \( i_{L_{\min}} > 0 \) and conduction in \( L_m \) is continuous.

(i) \[
\frac{V_o}{V_d} = \frac{N_2}{N_1} \frac{D}{1 - D} = \frac{3}{2} \frac{D}{1 - D}
\]

or \[
\frac{24}{10} = 1.5 \times \frac{D}{1 - D} \quad \therefore D = 0.615
\]

\[
\frac{V_o^2}{R} = 120 \quad \therefore R = \frac{V_o^2}{120} = 4.8 \Omega
\]

(ii) \[
L_m = \frac{(1 - 0.615)^2}{2 \times 100 \times 10^3} \times 4.8 \times \left( \frac{2}{3} \right)^2 = 1.58 \mu H
\]

(iii) \[
\frac{\Delta V_o}{V_o} \leq 0.01 \quad \therefore C = \frac{D}{R f_s \times 0.01} = 128 \mu F
\]
2. \( L_m = 1.58 \times 10^{-6}, \ D = 0.6, \ R = 12\Omega, \ f_s = 100kHz, \ V_d = 10V \)

\[
I_{L_{\text{min}}} = \frac{DV_d}{(1-D)^2} \left(\frac{N_2}{N_1}\right)^2 - \frac{DV_dT_s}{2L_m} = \frac{10 \times 0.615}{(1-0.615)^2} \left(\frac{1.5}{12}\right)^2 - \frac{0.615 \times 10}{2 \times 1.58 \times 10^{-6} \times 10^{-6}} \\
= 7.78 - 19.46 = -11.68A
\]

Since the magnetising current \( i_{L_m} \) cannot be negative, the conduction must be discontinuous.

From power balance,

\[
V_d I_d = \frac{V_o^2}{R}
\]

(i)

\[
.: V_o = \sqrt{RV_dI_d} = \sqrt{R \frac{V_o^2D^2}{2L_mf_s}} = V_dD \sqrt{\frac{R}{2L_mf_s}} = 10 \times 0.615 \sqrt{\frac{12}{2 \times 1.58 \times 10^{-6} \times 10^5}} = 37.9V
\]

which is higher than the desired output of 24V.
(ii) When $V_o$ is restored to 24V

$$P_o = \frac{V_o^2}{R} = \frac{24^2}{12} = 48W$$

$$P_d = V_d \times I_d = 48W$$

$$I_d = \frac{48}{V_d} = 4.8 \ A = \frac{V_d^2}{2L_m f_s}$$

$$D = \frac{4.8 \times 2 \times 1.58 \times 10^{-6} \times 100 \times 10^3}{10} = 0.389$$

Answer should be 0.427

This should be $2.4^2$ not 4.8

(iii) $P_{O,new} = \frac{24^2}{12} = 48W$

3. Since $N_3 = N_1$, $D_{max} = \frac{I}{I + \left(\frac{N_3}{N_1}\right)} = 0.5$

(i) $V_d = 43.2 - 52.8V$

$$\frac{N_2}{N_1} = \frac{V_o}{V_{d_{min}} \times D_{max}} = 0.232$$

For $V_d = 43.2 \ \text{V}$

$$\frac{N_2}{N_1} = \frac{5}{43.2 \times 0.5} = 0.232$$

If this turns ratio is selected and $V_d$ becomes 52.8 V, then

$$D = \frac{V_o}{V_d \times \left(\frac{N_2}{N_1}\right)} = \frac{5}{52.8 \times 0.232} = 0.408 \ \text{which is lower than} \ D_{max} = 0.5.$$
When this turns ratio is selected and $V_d$ becomes 43.2 V,

$$D = \frac{V_o}{V_d \times \left( \frac{N_2}{N_1} \right)} = \frac{5}{43.2 \times 0.189} = 0.612$$

which is higher than $D_{\text{max}} = 0.5$.

Therefore, we must select the smallest turns ratio $N_2/N_1 = 0.232$. Note that since only 0.2, and 0.25 are the nearest possible turn ratios, the above calculations must be repeated for these available turns ratios, to select the correct one. We will ignore this practical aspect in the following calculations.

(ii) $\frac{N_2}{N_1} = 0.232$, $P_o = 15W$, $I_{O_{\text{min}}} = I_{L_{\text{min}}} = \frac{15}{5} = 3A$

To operate the converter in the continuous conduction mode, $i_{L_{\text{min}}} \geq 0$. To ensure continuous conduction we should choose the lower output power, i.e., $P_o = 15W$.

At the boundary of continuous/discontinuous conduction:

$$i_{L_{\text{min}}} = I_L - \frac{\Delta i_L}{2} = 0$$

$$\therefore \frac{1}{2} \frac{(V_d \frac{N_2}{N_1} - V_o)}{L_{\text{min}}} DT_S = \frac{V_o (1 - D)}{2L_{\text{min}} f_s} = I_{O_{\text{min}}} = 3 \ A$$

When $V_d = 43.2$, and $N_2/N_1 = 0.232$, $D = \frac{5}{43.2 \times 0.232} = 0.5$

$$\therefore 3 = \frac{5(1 - 0.5)}{2L_{\text{min}} f_s}$$. Hence, $L_{\text{min}} = 4.16 \ \mu H$

When $V_d = 52.8 \ V \quad D = \frac{5}{52.8 \times 0.232} = 0.409$

$$\therefore L_{\text{min}} = 4.93 \mu H$$.

We must choose $L_{\text{min}} = 4.94 \mu H$ because $L = 4.16 \ mH$ will make $i_L$ discontinuous when $V_d = 52.5V$.

(iii) $D_{\text{max}} = 0.7$, $\frac{N_3}{N_i} = \frac{I}{D_{\text{max}}} - 1 = 0.429$

$$\therefore V_{T_{\text{max}}} = V_d + \frac{N_I}{N_3} V_d = 3.33 V_d$$
4.

(i) \[ V_o = V_a D \frac{N_2}{N_1} = 48 \times 0.4 \times \frac{1}{1.5} = 12.8V \]

\[ I_L = I_O = \frac{12.8}{10} = 1.28A \]

(ii) \[ \Delta I_L = \frac{V_o(1-D)T_s}{L} = \frac{12.8 \times (1-0.4)}{0.4 \times 10^{-3} \times 35000} = 0.55A \]

\[ \therefore i_{L_{\text{max}}} = 1.28 + \frac{0.55}{2} = 1.56A, \quad i_{L_{\text{min}}} = 1.28 - \frac{0.55}{2} = 1.01A \]

(iii) \[ \Delta V_O = \frac{(1-D)}{8LC_f^2} V_o = \frac{12.8 \times (1-0.4)}{8 \times 0.4 \times 10^{-3} \times 100 \mu F \times (35)^2 \times 10^6} = 19.6mV \]

(iv) The primary current is the sum of the magnetising current and the reflected current from the secondary. The peak magnetising current:

\[ i_{L_{\text{max}}} = \frac{V_a DT_s}{L_m} = \frac{48 \times 0.4}{5 \times 10^{-3} \times 35000} = 0.11A \]

\[ \therefore i_{p_{\text{max}}} = \frac{N_2}{N_1} i_{L_{\text{max}}} + i_{L_{m_{\text{max}}}} = 1.56 \times \frac{1}{1.5} + 0.11 = 1.15A \]

(v) Time taken by the magnetising current to fall to zero is given by:

\[ t_i = DT_s \frac{N_1}{N_i} = \frac{0.4}{35000} \times 1 = 11.4 \mu s \]

Also, \( DT_s = 11.4 \mu \text{sec} \)

\[ \therefore i_{L_m} \text{ reaches zero } 11.4 + 11.4 = 22.8 \mu s \text{ into each switching period.} \]

Since \( T_s = 28.57 \mu s \), the transformer resets in each cycle.
5. The output voltage waveform \( v_o \) is

\[
\begin{align*}
\omega t = 0 & \quad \text{\( V_d \)} \\
-\pi & \quad \omega t = 0 \\
\pi & \quad \text{\( \delta \)}
\end{align*}
\]

By taking the \( \omega t = 0 \) reference in the middle of the positive pulse as indicated, since \( f(\theta) = -f(-\theta) \),

\[
a_n = \frac{1}{\pi} \int_0^{2\pi} V_d \cos(n \omega t) d(\omega t) = \frac{4V_d}{\pi} \int_0^{\frac{\delta}{\pi}} \cos(n \omega t) d(\omega t)
\]

\[
= \frac{4V_d}{n\pi} \sin \frac{n\delta}{2}
\]

Note that, \( \delta = \pi \). The output voltage \( v_o \) is given by

\[
v_o = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V_d}{n\pi} \sin \frac{n\delta}{2} \cos(n \omega t)
\]

(i) \( V_1 = \text{rms value of the fundamental} = \frac{4V_d}{\sqrt{2}\pi} \)

\( V_3 = \text{rms value of the 3}^{\text{rd}} \text{ harmonic} = \frac{4V_d}{3\sqrt{2}\pi} \)

\( V_5 = \text{rms value of the 5}^{\text{th}} \text{ harmonic} = \frac{4V_d}{5\sqrt{2}\pi} \)

\[
\vdots
\]

and so on

(ii) For the required fundamental rms voltage is 240 V,
\[ V_I = \frac{4V_d}{\sqrt{2} \pi} = 240 \text{ V} \]
\[
\therefore V_d = \frac{240 \times \sqrt{2} \pi}{4} = 266.4 \text{ V}
\]

(iii) Expressions for the load current waveform can be obtained in three ways.

**Method 1:**
\[
i_1 = \frac{4V_d}{\pi |Z_1|} \cos \left\{ \omega t - \tan^{-1} \left( \frac{\omega L}{R} \right) \right\}
\]
\[
i_3 = \frac{4V_d}{3\pi |Z_3|} \cos \left\{ 3\omega t - \tan^{-1} \left( \frac{3\omega L}{R} \right) \right\}
\]
\[
i_5 = \frac{4V_d}{3\pi |Z_5|} \cos \left\{ 5\omega t - \tan^{-1} \left( \frac{5\omega L}{R} \right) \right\}
\]
\[\ldots\]
\[\text{and so on, where } |z_n| = \sqrt{R^2 + (n\omega L)^2}\]
\[i_o = i_1 + i_3 + i_5 + \ldots\]

The number of harmonics to be included in the above sum is determined by the significance of the relative contribution of the highest harmonic term.

**Method 2:**

Solution for the load current \(i_o\) in closed form is obtained by finding the steady-state maximum and minimum values of the load current. Using the closed-form solution covered in lecture

\[I_I = I_{min} = \frac{V}{R} \left( 1 - e^{\frac{\theta}{L}} \right) \quad \text{where } t_1 = 10 \text{ msec, the ON-time for T1 and T2 for 50 Hz output.}\]

Given: \(R = 9 \ \Omega, \ L = 9/314 = 0.0287 \ \text{H}\)

\[I_I = I_{min} = \frac{266.4 \cdot 1 - e^{\frac{\theta}{9}}}{9} \cdot \frac{\theta^{\pi}}{1 + e^{\frac{\theta}{9}}} = -27.13 \text{ A}\]

\[I_{max} = 27.13 \text{ A}\]

*Solution to Tutorial 5 – Isolated DC-DC Converters and Inverters*
For $0 < t < 0.01$ sec

$$i = \frac{V_d}{R} + \left(-27.13 - \frac{V_d}{R}\right) e^{\frac{R}{L} t}$$

For $0.01 < t < 0.02$ sec

$$i = -\frac{V_d}{R} + \left(27.13 + \frac{V_d}{R}\right) e^{\frac{R}{L} t}$$

Method 3:

The solution of the differential equation of the load current can be carried out for each half cycle, starting from $t = 0$ sec, with the initial value for the load current assumed to be zero. At the end of each half cycle, the value of current becomes the initial current into the next half cycle which is a better estimate. The solution is repeated for subsequent half cycles until it is found that $i_{\text{max}}$ is equal to $i_{\text{min}}$, albeit for its sign. This iterative process is very easily implemented on a computer.

(iv)

$$P_o = \frac{1}{0.01} \int_0^{0.01} i^2 R \, dt \quad \text{..........(1)}$$

$$= \sum_{n=1,3,5,...} V_n I_n \cos \phi_n \quad \text{..........(2)}$$
where \( V_n = \frac{4V_n}{\sqrt{2} n\pi} \); \( I_n = \frac{V_n}{\sqrt{R^2 + (n\omega L)^2}} \); \( \phi_n = \tan^{-1}\left(\frac{n\omega L}{R}\right) \)

6.

(i) \( V_i = \frac{4V_d}{n\pi \sqrt{2}} \sin n\delta = \frac{4 \times 380}{\pi \sqrt{2}} = 342.34 \) V (rms)

\[
I_{\text{min}} = \frac{V_d}{R} \left(1 - e^{-\left(\frac{R}{L}\right)t_1}\right) \left(1 + e^{-\left(\frac{R}{L}\right)t_1}\right)
\]

where \( t_1 = 0.01 \) sec.

\[= -31.19 \text{ A} \]

For \( 0 < t < 0.01 \) sec

\[i_o = \frac{V_d}{R} - \left(\frac{V_d}{R} - I_{\text{min}}\right) e^{-\left(\frac{R}{L}\right)t} = 38 - 69.9e^{-314t} \text{ A} \]

(ii) Quasi-square-wave output:

\[V_{\text{rms}} = \frac{4V_d}{\pi \sqrt{2}} \sin \left(\frac{90^\circ}{2}\right) = 242 \text{ V} \]

The expression for the load current \( i_o \) is obtained from

\[L \frac{di}{dt} + Ri = V \]

\(2.5 < t < 7.5 \) msec

By taking \( t_1 = 0 \) at \( t = 2.5 \) msec,
\[ i_o = 38 - 38e^{-314t} \]

\[ i_o = 38 - 38e^{-314t} = 38 - 38e^{-314 \times 0.005} = 30.09 \text{ A} \]

Note that current during first 2.5 msec is zero.

**7.5 < t < 12.5 msec**

Current falls exponentially while freewheeling locally through a diode and a switch. By taking time \( t_2 = 0 \) at \( t = 7.5 \) msec,

\[ i_o = 30.09e^{-314t} \text{ A} \]

At \( t = 12.5 \) msec, \( i_o = 30.09e^{-314t} = 30.09e^{-319 \times 0.005} = 6.26 \text{ A} \)

**12.5 < t < 17.5 msec**

Load current \( i_o \) rises negatively when \(-380 \text{ V}\) applies across the load. By taking \( t_3 = 0 \) at \( t = 12.5 \) msec,

\[ i_o = -38 + (38 + 6.26)e^{-314t} \text{ A} \]

At \( t = 17.5 \) msec, \( i_o = -38 + (38 + 6.26)e^{-314t} = -38 + (38 + 6.26)e^{-314 \times 0.005} = -28.79 \text{ A} \)

**17.5 < t < 22.5 msec**

Currents falls exponentially, while freewheeling through a diode and a switch. By taking time \( t_4 = 0 \) at \( t = 17.5 \) msec,

\[ i_o = -28.79e^{-314t} \text{ A} \]

\( i_o \) at 22.5 msec is \( i_{o22.5} = -28.79e^{-314 \times 0.005} = -5.99 \text{ A} \)

**22.5 < t < 27.5 msec**

\[ i_o = 38 - (38 + 5.99)e^{-314t} \text{ A} \]

\[ i_{o27.5} = 38 - (38 + 5.99)e^{-314 \times 0.005} = 28.84 \text{ A} \]

**27.5 < t < 32.5 msec**

\[ i_o = 28.84e^{-314t} \]
\[ i_{0.32.5} = 28.84 e^{-314 \times 0.905} = 6 \text{ A} \]
\[ 32.5 < t < 37.5 \text{ msec} \]

\[
i_o = 38 - (38 + 6) e^{-314 \times t_f} = 28.84
\]

\[
i_{o37.5} = -38 + (38 + 6) e^{-314 \times 0.005} = -28.84 \text{ A}
\]

which is the same as at 27.5 msec. The output current settles after this time.

(iii) \[
V_{ol} = V_{ol\max} \cos \frac{\alpha}{2}
\]

\(V_{ol\max}\) occurs for the square-wave output when \(\alpha = 0^\circ\), i.e., \(\delta = 180^\circ\).

Thus, \(V_{ol\max} = \frac{4V_d}{\pi \sqrt{2}} = 342.34 \text{ V}\)

\[V_{ol} = 200 = 342.34 \cos \frac{\alpha}{2}\]

\[\therefore \alpha = 108.5^\circ\]

(iv) \[\alpha = 108.5^\circ, \quad \delta = 180 - \alpha = 180 - 108.5 = 71.6^\circ\]

\[V_3 = \frac{4V_d}{3\pi \sqrt{2}} \sin \frac{3\delta}{2} = 108.98 \text{ V}; \quad V_5 = \frac{4V_d}{5\pi \sqrt{2}} \sin \frac{5\delta}{2} = 1.49 \text{ V}\]
7. The waveform has quarter-wave symmetry.

By taking $\omega t = 0$ at the center of the positive pulse,

$$a_n = \frac{4}{\pi} \int_{-\delta/2}^{\delta/2} V_d \cos(n \omega t) dt = \frac{4V_d}{n\pi} \left[ \sin(n \omega t) \right]_{-\delta/2}^{\delta/2} = \frac{4V_d}{n\pi} \sin \left( \frac{n\delta}{2} \right)$$

for $n = 1, 3, 5, \ldots$

When the center of the pulse is displaced from the $\omega t = 0$ axis by an arbitrary angle $\alpha$ as shown below, the symmetry conditions do not apply and, ordinarily, we have to find both $a_n$ and $b_n$ coefficients of the Fourier series representing the waveform. The foregoing result for $a_n$ can still be used without finding $b_n$ explicitly.

Note that the harmonics of the quasi-square waveform will be displaced by angle $n\alpha$ and that their amplitudes remain unchanged. The harmonic can be represented by their phasors in terms of their rms value and phase angle $n\alpha$. Thus

$$V_n = \frac{4V_d}{n\pi \sqrt{2}} \sin \left( \frac{n\delta}{2} \right) \angle n \alpha$$

$$= \frac{4V_d}{n\pi \sqrt{2}} \sin \left( \frac{n\delta}{2} \right) \left[ \cos n\alpha + j \sin n\alpha \right]$$
The instantaneous values of the harmonics are given by

\[ v_n = \frac{4V_d}{n\pi} \sin \left( \frac{n\delta}{2} \right) \cos n(\omega t + \alpha) \]

and

\[ v_o = \sum_{i=3,5,\ldots}^{\infty} \frac{4V_d}{n\pi} \sin \left( \frac{n\delta}{2} \right) \cos n(\omega t + \alpha) \]

For the PWM waveform given below

The voltage pulses P1 and P3 are of the same width. The switching angle \( \alpha_1 \) and \( \alpha_2 \) define the widths of all three voltage pulses in each half cycle. The \( \omega t = 0 \) reference is assumed at the center of pulse P2. Thus

For pulse P1: \( \delta_1 = \alpha_1 \); displacement from the reference = \( \frac{\pi}{2} - \frac{\alpha_1}{2} \) leading

\[ V_{nP1} = \frac{4V_d}{n\pi \sqrt{2}} \sin \left( \frac{n\alpha_1}{2} \right) \angle n \left( \frac{\pi}{2} - \frac{\alpha_1}{2} \right) \]

\[ = \frac{4V_d}{n\pi \sqrt{2}} \sin \left( \frac{n\alpha_1}{2} \right) \left\{ \cos n \left( \frac{\pi}{2} - \frac{\alpha_1}{2} \right) + j \sin n \left( \frac{\pi}{2} - \frac{\alpha_1}{2} \right) \right\} \]

For pulse P2: \( \delta_2 = \pi - 2\alpha_2 \); displacement from reference = 0

\[ V_{nP2} = \frac{4V_d}{n\pi \sqrt{2}} \sin \left[ n \left( \frac{\pi - 2\alpha_2}{2} \right) \right] \angle 0 = \frac{4V_d}{n\pi \sqrt{2}} \sin \left[ n \left( \frac{\pi - 2\alpha_2}{2} \right) \right] \]

For pulse P3; \( \delta_3 = \alpha_1 \); displacement from reference = \( \frac{\pi}{2} - \frac{\alpha_1}{2} \) lagging
\[ V_{nR3} = \frac{4V_d}{n\pi\sqrt{2}} \sin \left( \frac{n\alpha_1}{2} \right) \angle -n \left( \frac{\pi}{2} - \frac{\alpha_1}{2} \right) \]

\[ = \frac{4V_d}{n\pi\sqrt{2}} \sin \left( \frac{n\alpha_1}{2} \right) \left\{ \cos n \left( \frac{\pi}{2} - \frac{\alpha_1}{2} \right) - j \sin n \left( \frac{\pi}{2} - \frac{\alpha_1}{2} \right) \right\} \]

For \( n = 1 \)

\[ V_1 = \frac{4V_d}{\pi\sqrt{2}} \left[ \sin \left( \frac{\alpha_1}{2} \right) \left\{ \cos \left( \frac{\pi}{2} - \frac{\alpha_1}{2} \right) + j \sin \left( \frac{\pi}{2} - \frac{\alpha_1}{2} \right) \right\} + \sin \left( \frac{\pi - 2\alpha_2}{2} \right) + \right] \]

\[ + \sin \left( \frac{\alpha_1}{2} \right) \left\{ \cos \left( \frac{\pi}{2} - \frac{\alpha_1}{2} \right) - j \sin \left( \frac{\pi}{2} - \frac{\alpha_1}{2} \right) \right\} \]

\[ = \frac{4V_d}{\pi\sqrt{2}} \left[ 2 \sin^2 \frac{\alpha_1}{2} + \cos \alpha_2 \right] \]

\[ = \frac{4V_d}{\pi\sqrt{2}} \left[ 1 - \cos \alpha_1 + \cos \alpha_2 \right] = 0.8V_d \quad \text{.................(1)} \]

For \( n = 3 \)

\[ V_3 = \frac{4V_d}{3\pi\sqrt{2}} \left[ \sin \left( \frac{3\alpha_1}{2} \right) \left\{ \cos 3 \left( \frac{\pi}{2} - \frac{\alpha_1}{2} \right) + j \sin 3 \left( \frac{\pi}{2} - \frac{\alpha_1}{2} \right) \right\} + \sin 3 \left( \frac{\pi - 2\alpha_2}{2} \right) + \right] \]

\[ + \sin \left( \frac{3\alpha_1}{2} \right) \left\{ \cos 3 \left( \frac{\pi}{2} - \frac{\alpha_1}{2} \right) - j \sin 3 \left( \frac{\pi}{2} - \frac{\alpha_1}{2} \right) \right\} \]

\[ = \frac{4V_d}{3\pi\sqrt{2}} \left( \sin 3\alpha_1 - 1 - \cos 3\alpha_2 \right) = 0 \quad \text{.................(2)} \]

Equations (1) and (2) must be solved simultaneously, in order to obtain \( \alpha_1 \) and \( \alpha_2 \). These equations are nonlinear (transcendental) and can not be solved by linear methods.
8. This problem is based on the analysis of PWM waveforms by using the impulse or delta functions which was included in the lecture notes.

For $k = 5$

\[(i) \quad B_n = 0.4mV_d \left[ \sin \frac{\pi}{10} \sin \frac{n\pi}{10} + \sin \frac{3\pi}{10} \sin \frac{3n\pi}{10} + \sin \frac{5\pi}{10} \sin \frac{5n\pi}{10} + \right. \\
+ \sin \frac{7\pi}{10} \sin \frac{7n\pi}{10} + \sin \frac{9\pi}{10} \sin \frac{9n\pi}{10} \left. \right] \]

For $n = 1$

\[B_1 = mV_d\]

\[V_1 = \frac{mV_d}{\sqrt{2}}\]

Given, $V_1 = 100$ V, $V_d = 200$ V

\[\therefore m = \frac{100\sqrt{2}}{200} = 0.707\]

For $n = 3, 5, 7$

\[B_n = 0\]

Note that the output voltage harmonics for this unipolar switched inverter (three-level output) are at frequencies of $2m_j r \pm 1$ where $r = 1, 2, 3, \ldots$

\[(ii) \quad B_g = 0.4 \times 0.707 \times 200 \left[ \sin \frac{\pi}{10} \sin \frac{9\pi}{10} + \sin \frac{3\pi}{10} \sin \frac{3\times9\pi}{10} + \sin \frac{5\pi}{10} \sin \frac{5\times9\pi}{10} + \right. \\
+ \sin \frac{7\pi}{10} \sin \frac{7\times9\pi}{10} + \sin \frac{9\pi}{10} \sin \frac{9\times9\pi}{10} \left. \right] \]

\[= 141.4\]

\[\therefore V_g = \frac{|B_g|}{\sqrt{2}} = 100 \text{ V (RMS)}\]

\[B_{11} = 0.4 \times 0.5 \times 200 \left[ \sin \frac{\pi}{10} \sin \frac{11\pi}{10} + \sin \frac{3\pi}{10} \sin \frac{3\times11\pi}{10} + \sin \frac{5\pi}{10} \sin \frac{5\times11\pi}{10} + \right. \\
+ \sin \frac{7\pi}{10} \sin \frac{7\times11\pi}{10} + \sin \frac{9\pi}{10} \sin \frac{9\times11\pi}{10} \left. \right] \]

\[= -141.4\]
\[ V_{11} = \frac{|B_{11}|}{\sqrt{2}} = 100 \ \text{V (RMS)} \]

(iii)

\[ I_g = \frac{V_o}{R + j9\omega L} = \frac{100}{\sqrt{10^2 + \left(9 \times 314 \times 0.02\right)^2}} = \frac{100}{57.39} = 1.7426 \ \text{A (RMS)} \]

\[ I_{11} = \frac{V_{11}}{R + j11\omega L} = \frac{100}{\sqrt{10^2 + \left(11 \times 314 \times 0.02\right)^2}} = \frac{100}{69.8} = 1.4328 \ \text{A (RMS)} \]

\[ I_{h,RMS} = \sqrt{I_g^2 + I_{11}^2} = \sqrt{1.7426^2 + 1.4328^2} = 2.256 \ \text{A neglecting higher order terms} \]

(iv)

\[ \text{THD of } V_o = \sqrt{\sum_{n=11}^{\infty} V_n^2} \]

\[ \text{THD of } V_o = \frac{1.414 \times 100\%}{1} = 141.4\% \]

This result does not reveal much and is meaningless. The high THD is due to the fact the harmonic components are large. However these are at high frequencies, and are easily filtered out. A more meaningful result is obtained when harmonic currents in the load are considered.

\[ I_l = \frac{V_l}{\sqrt{R^2 + (\omega L)^2}} = \frac{100}{\sqrt{10^2 + \left(314 \times 0.02\right)^2}} = \frac{100}{11.81} = 8.47 \ \text{A} \]

\[ \text{THD of } I_o = \frac{\sqrt{I_g^2 + I_{11}^2}}{I_l} = \frac{\sqrt{1.7426^2 + 1.4328^2}}{8.47} = 0.2663 = 26.63\% \]

If the THD in the \( I_o \) is to be lower than 26.63\%, the number of pulses per half cycle, \( k \), is to be increased appropriately.
(i). The output voltage waveform is

(ii) By taking the $\alpha = 0$ reference as shown above and using the results of 3,

$$V_n = \frac{4V_d}{n\pi\sqrt{2}} \left[ \sin \frac{n\delta_1}{2} + \sin \frac{n\delta_2}{2} \{\cos n\alpha + j\sin n\alpha\} + \sin \frac{n\delta_2}{2} \{\cos n\alpha - j\sin n\alpha\} \right]$$

$$= \frac{0.9V_d}{n} \left[ \sin \frac{n\delta_1}{2} + 2 \sin \frac{n\delta_2}{2} \cos n\alpha \right]$$

$$= \frac{0.9V_d}{n} \left[ \sin (n \times 30^\circ) + 2 \sin (n \times 10^\circ) \times \cos (n \times 60^\circ) \right]$$

$$V_3 = \frac{0.9V_d}{3} \left[ \sin (3 \times 30^\circ) + 2 \sin (3 \times 10^\circ) \times \cos (3 \times 60^\circ) \right]$$

$$= 0.3 \left[ 1 + 2 \times \frac{1}{2} \times (-1) \right] = 0$$

Hence, the third harmonic in the output is eliminated!

For high power inverter circuits, where the switching frequency is quite low, the inverse problem of finding the switching angles so that certain low order output voltage harmonics are eliminated, is often faced.

(iii) $\quad V_I = \frac{0.9V_d}{1} \left[ \sin 30^\circ + 2 \sin 10^\circ \times \cos 60^\circ \right]$

$$= 0.9V_d \left( 0.5 + 0.17 \right) = 0.606V_d$$

The rms value of the output voltage waveform $v_o$ is
\[ V_o = \frac{1}{\pi} \int_{20^\circ}^{40^\circ} V_d^2 d(\omega t) + \int_{60^\circ}^{120^\circ} V_d^2 d(\omega t) + \int_{140^\circ}^{160^\circ} V_d^2 d(\omega t) \]

\[ = V_d \sqrt{\frac{1}{\pi} \times \frac{100}{180} \times \pi} \]

\[ = 0.745V_d \]

\[ \therefore \quad THD = \sqrt{\frac{V_o^2 - V_i^2}{V_i}} = \frac{\sqrt{0.745^2 - 0.606^2}}{0.606} = 0.715 = 71.5\% \]

10.

Note that the line-line voltage waveforms are quasi-square, with amplitude of \( V_d \) and \( \delta = 120^\circ \). For the \( Y \)-connected load, the line-neutral voltage waveforms are stepped square, with amplitude of \( \frac{2}{3}V_d \). In order to find the line-neutral voltage, the voltage of the neutral point must first be found by noting the states of the switches and by assuming a purely resistive load. Note that when two of the top switches and one of the bottom switches are ON, the voltage of the neutral point is \( \frac{2}{3}V_d \). When one of the top switches and two of the bottom switches are ON, the voltage of the neutral point is \( \frac{1}{3}V_d \). The line-neutral voltage is found by subtracting the voltage of the...
neutral point from the line voltage. The line-line voltages are found more easily, simply by noting the states of the switches and subtracting the voltage of one line from another.
During \(0 < \omega t < \pi/3\)

Top switches T1 & T5 and bottom switch T6 are ON.

\[ V_d = iR + 2iR \; ; \; \therefore \; i = \frac{V_d}{3R} \]

Potential of the neutral point with respect to the –ve dc rail is \(2i \times R\), i.e., \(\frac{2V_d}{3}\).

Since load terminals \(a\) and \(b\) are at the same potential, \(v_{ac} = 0\).

The line-line voltages are:

\[ v_{ab} = V_d; \; v_{bc} = -V_d; \; v_{ca} = 0 \]

The line-neutral voltages are:

\[ v_{an} = V_d - v_n = V_d - \frac{2}{3}V_d = \frac{1}{3} V_d \; ; \]

\[ v_{cn} = \frac{1}{3} V_d \; ; \; v_{bn} = -\frac{2}{3} V_d \]

During \(\pi/3 < \omega t < 2\pi/3\)

Top switch T1 and bottom switches T2 and T6 are ON.

\[ V_d = 2i \times R + i \times R \; . \; \therefore \; i = \frac{V_d}{3R} \]

\[ v_n = \frac{1}{3} V_d \] with respect to the –ve dc rail.

The line-line voltages are:

\[ v_{ab} = V_d; \; v_{bc} = 0; \; v_{ca} = -V_d \]

The line-neutral voltages are:

\[ v_{an} = V_d - v_n = V_d - \frac{1}{3}V_d = \frac{2}{3} V_d \]

\[ v_{bn} = -\frac{1}{3} V_d \; ; \; v_{cn} = -\frac{1}{3} V_d \]
During $2\pi/3 < \omega t < \pi$

T1, T3 and T2 ON

\[ v_n = \frac{2}{3} V_d \] with respect to the –ve dc rail.

The line-line voltages are:

\[ v_{ab} = 0, \quad v_{bc} = V_d, \quad v_{ca} = -V_d. \]

The line – neutral voltages are:

\[ v_{an} = \frac{1}{3} V_d, \quad v_{bn} = \frac{1}{3} V_d, \quad v_{cn} = -\frac{2}{3} V_d. \]

The voltage waveforms in $\pi < \omega t < 2\pi$ are opposite polarity of the voltage in $0 < \omega t < \pi$. Furthermore, being a balanced three-phase system, all line voltages are of equal amplitude, as are line-neutral voltages. They also are displaced by ±120° between phases.

From observation of the waveforms, the rms values are:

\[ v_{ab} = \frac{4V_d}{n\pi} \sum_{n=1,3,5,\ldots}^{\infty} \sin n \times 120^\circ \cos n \left( \omega t - \frac{\pi}{3} \right) \]

\[ V_{1,1} = \frac{4V_d}{\pi \sqrt{2}} \sin 60^\circ = 0.78V_d = 156 \text{ V} \]

\[ V_{5,1} = -31.21 \text{ V} \]

\[ V_{7,1} = 22.29V; \quad V_{11,1} = -14.19V; \quad V_{13,1} = 12V; \quad V_{17,1} = 9.18 \text{ V} \]

\[ V_{0,1} = \sqrt{\frac{1}{\pi} \int_0^{2\pi/3} V_d^2 d(\omega t)} = 0.8165V_d \text{ V} \]

\[ V_{1,1} = \frac{V_{1,1}}{\sqrt{3}} = 0.45V_d = 90 \text{ V} \]

\[ V_{5,1} = -18.02 \text{ V}; \quad V_{7,1} = 13.22 \text{ V}; \quad V_{11,1} = 8.19 \text{ V}; \quad V_{13,1} = 6.93 \text{ V}; \quad V_{17,1} = 5.3 \text{ V} \]