1.3 Thermal Transients

As we have seen in section 1.2, the basic equation governing heat transfer and temperature rise in an item of electrical equipment subject to a step change in load is the following equation, with $d\theta$ and $dt$ being incremental changes in temperature and time respectively:

$$P dt = m c d\theta + A h (\theta - \theta_o) dt$$

where:

- $P$ = power dissipation (loss generation) level in the equipment
- $m$ = total equipment mass
- $c$ = specific heat of the equipment material
- $h$ = overall heat dissipation coefficient from the equipment surface
- $A$ = surface area available for heat dissipation from equipment
- $\theta$ = surface temperature
- $\theta_o$ = ambient temperature
The loss $P$ includes all forms of heat loss generation in the equipment. These will be one or more of the mechanisms discussed earlier.

For constant levels of the heat loss $P$, it is shown in the Appendix that the general solution of the above equation for the temperature rise of the surface above ambient is:

$$\theta(t) - \theta_o = \frac{P}{Ah} \left[1 - e^{-t/\tau}\right]$$

$$= \Delta\theta_{ss} \left[1 - e^{-t/\tau}\right]$$

where $\Delta\theta_{ss} = P/(Ah)$ is the steady state temperature rise above the ambient and the initial condition is $\theta(0) = \theta_o$.

$$\Delta\theta_{ss} = \theta_{ss} - \theta_o$$

and $\tau$ is the thermal time constant:

$$\tau = \frac{mc}{Ah}$$

Thus the temperature rise variation is exponential as shown in the following diagram.
In fact, using the appropriate initial condition in the solution, it can be shown that, irrespective of the starting temperature, for a given $P$, $A$ and $h$ values, $\theta_{ss}$ is the final temperature for any starting point, such as $\theta_1$ for example.

$$\theta(t) - \theta_1 = (\theta_{ss} - \theta_1)(1 - e^{-t/\tau})$$

Even if $\theta_1 > \theta_{ss}$ or if $\theta_1 < \theta_o$, the final temperature is still $\theta_{ss}$ for the given $P$, $A$ and $h$ values.
**Cooling of Equipment**

When the power source is disconnected from the equipment, $P = 0$ and the general equation then becomes:

$$mcd\theta + Ah(\theta - \theta_o)dt = 0$$

The solution of this equation is (taking the starting temperature $\theta(0)$ as $\theta_{ss}$):

$$\theta(t) - \theta_o = (\theta_{ss} - \theta_o)e^{-t/\tau}$$

If the initial temperature is $\theta_1$, the solution is:

$$\theta(t) - \theta_o = (\theta_1 - \theta_o)e^{-t/\tau}$$

If the final temperature condition is $\theta_2 > \theta_o$, such as may happen with a transformer with load removed but still energised with core losses keeping the temperature above ambient,

$$\theta(t) - \theta_2 = (\theta_{ss} - \theta_2)e^{-t/\tau}$$

The general variations of these situations are shown below.
It can be seen that the two important quantities from the above analysis are the steady state temperature rise $\theta_{ss}$ and the thermal time constant $\tau$, which are defined as below:

**Steady state temperature rise:**

$$\Delta \theta_{ss} = \frac{P}{Ah}$$

This is the ratio of the power loss to the total power dissipation rate of the equipment.

**Equipment time constant:**

$$\tau = \frac{mc}{Ah}$$

This is the ratio of the equipment thermal capacity to the total power dissipation rate of the equipment.

**Typical values of equipment thermal time constants**

**Overhead lines and busbars**

With their relatively low mass (because they have no insulation) and hence low thermal capacity, and also with relatively good heat dissipation rates, overhead lines have quite short time constants (perhaps 15 –30 minutes or so).

**Example:** 430 mm$^2$ ACSR conductor (330 kV overhead line)

$$\tau \approx 20 \text{ minutes}$$
Cables and transformers

These items have substantial insulation and thus very large mass and thermal capacity and, in general, less efficient dissipation mechanisms. As a result they have high value time constants (many hours).

Examples:

1000 mm$^2$ copper conductor, paper-oil insulated cable
\[ \tau \cong 1.3 \text{ hours} \]

120 MVA, 275 kV transmission transformer
\[ \tau \cong 2 \text{ hours (with forced convection)} \]
\[ \tau \cong 7.5 \text{ hours (with natural convection)} \]

The thermal time constants are very important in rating calculations particularly for:

- Cyclic rating (rating for loading with cyclic variation, the duration of the cycle usually being one day)
- Short time rating
- Emergency rating

The calculation is very complex given the numbers of possible components and their different mass and thermal capacity in large equipment. For transformers there are available empirical equations that can be used. The Australian Standard on transformers AS2374.7-1997 gives such equations and typical values of the time constants for oil-immersed power transformers.
Starting from cold (de-energised), if a transformer has a continuous load applied then the time constant is given by:

$$\tau = \frac{C \theta}{W} \quad \text{(in hours)}$$

where:
- $\theta$ = top-oil temperature rise for steady-state conditions at the specified load.
- $W$ = total transformer losses at initial temperature (W).
- $C$ = thermal capacity of the transformer (Wh/°C)

(a) for natural oil cooling
- $C = 0.132 \times \text{mass of core and coils, in kg} + 0.088 \times \text{mass of tank, in kg} + 0.35 \times \text{volume of oil, in litres}$

(b) for forced oil cooling
- $C = 0.132 \times \text{mass of core and coils, in kg} + 0.088 \times \text{mass of tank, in kg} + 0.51 \times \text{volume of oil, in litres}$

<table>
<thead>
<tr>
<th>Method of cooling</th>
<th>Rating kV.A</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>≤ 12.5</td>
<td>37</td>
<td>73</td>
<td>≤ 123</td>
</tr>
<tr>
<td>ONAN</td>
<td>≤ 50</td>
<td>3.5</td>
<td>5.0</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>&gt; 50 ≤ 250</td>
<td>3.0</td>
<td>4.0</td>
<td>5.0</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>&gt; 250 ≤ 1000</td>
<td>2.75</td>
<td>3.5</td>
<td>4.0</td>
<td>5.0</td>
</tr>
<tr>
<td></td>
<td>&gt; 1000 ≤ 10000</td>
<td>2.5</td>
<td>2.75</td>
<td>3.5</td>
<td>4.0</td>
</tr>
<tr>
<td></td>
<td>&gt; 10000</td>
<td>2.5</td>
<td>2.75</td>
<td>3.0</td>
<td>3.5</td>
</tr>
<tr>
<td>ONAF</td>
<td>≤ 10000</td>
<td>2.25</td>
<td>2.5</td>
<td>2.75</td>
<td>3.0</td>
</tr>
<tr>
<td>OFAN</td>
<td>&gt; 10000 ≤ 30000</td>
<td>2.0</td>
<td>2.0</td>
<td>2.25</td>
<td>2.5</td>
</tr>
<tr>
<td>OFDAN</td>
<td>≤ 30000</td>
<td>1.75</td>
<td>1.75</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>OFAF</td>
<td>≤ 10000</td>
<td>1.75</td>
<td>1.75</td>
<td>2.0</td>
<td>2.5</td>
</tr>
<tr>
<td>OFDAF</td>
<td>&gt; 10000 ≤ 30000</td>
<td>1.5</td>
<td>1.5</td>
<td>1.75</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td>&gt; 30000</td>
<td>1.25</td>
<td>1.25</td>
<td>1.25</td>
<td>1.5</td>
</tr>
</tbody>
</table>
**Equipment Rating**

To determine the various thermal rating properties of equipment we need to know the following information about the equipment and the ambient environment in which it is to be operating.

1. \( P \) power loss – including all contributing loss mechanisms
2. \( mc \) thermal capacity – including all component parts
3. \( A \) surface area for dissipation
4. \( h \) heat dissipation coefficient – including all mechanisms
5. \( \theta_{max} \) maximum permissible temperature of the equipment
6. \( \theta_0 \) the ambient temperature

Items 5 and 6 above are particularly important for the rating determinations.

\( \theta_{max} \) is (generally, but not always) determined by the properties of the equipment insulation and the ageing rate.

\( \theta_0 \) is also very important (and variable): it can depend on many factors, including:

(i) the site location (c.f. Darwin and Hobart)
(ii) the enclosure (if any) (e.g. a box or a fuse enclosure or open air)
$\theta_0$ can vary extensively in any one location over the year or even over a day: the value used must thus be chosen carefully. In most cases, a *weighted average ambient temperature* is used and this is determined, for the calculation purposes, from maps such as the one shown below, which gives isotherm plots of standard weighted ambient temperatures all over Australia.

[Weighted annual mean ambient air temperature. [AS3768-1990 Guide to the effects of temperature on electrical equipment]]

Australian Standards generally state the temperature rise limits which must not be exceeded with a maximum ambient temperature of 40°C. The weighted annual mean ambient air temperature is taken as 20°C.
This use of just a single value to cover all times is obviously not as accurate as it could be to provide suitable ratings at all times during the year and even for the daily variation. In modern power systems, with the powerful computing facilities now available it is now more likely that the ratings will be determined as a **probabilistic rating**, where the average temperatures over the day and year can be used to determine an accurate rating at any specific time of the day and season. In some cases, equipment environment will be continuously monitored for ambient temperature and this will be transmitted to the central site where the true rating under the current environment conditions can be continually updated and utilised.

In the simple rating calculation model using the average temperature, there are **two potential inaccuracies** caused by the uncertainty in the ambient temperature of the equipment. These are the possible variations of the actual ambient from the average value used in the calculation and the possibility that the equipment may be confined in an enclosure with the fluid in the enclosure then heated to a higher temperature than the actual true ambient outside.

In both cases there are correction factors available that can be used to modify the basic steady state thermal rating calculation in such circumstances. The method of use of the equations is detailed below. Following that are extracts from Australian Standards giving details of how the above can be used in ratings calculations.
(i) Change (increase) in ambient temperature

When rating calculations for particular equipment items have been determined using specified ambient temperatures and the actual true ambient differs from that specified value, the Australian Standard AS3768 provides formulae for determining correction factors to the steady state ratings to take account of the changed ambient temperature. This correction factor may be necessary for operation during different seasons of the year or in quite different locations to that used for the ratings determination.

For example, if the thermal rating calculation is done at 40°C, obtaining \( I_{th} \) for a maximum permissible equipment temperature of \( T_n \), then for a different (higher) ambient temperature \( T_a \), the new thermal rating \( I_{th}' \) is given by:

\[
I_{th}' = K I_{th} = \left( \frac{T_n - T_a}{T_n - 40} \right)^{0.6} \times I_{th}
\]

For example if \( T_n = 100°C \), \( T_a = 45°C \), then the de-rating factor is \( K = 0.95 \). For other values of \( T_a \) and \( T_n \), the results are given in the table below.

<table>
<thead>
<tr>
<th>Ambient Temperature ( (T_a) ) (^°C)</th>
<th>Correction coefficient ( (K) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Maximum permissible temperature ( (T_n) ), (^°C)</td>
</tr>
<tr>
<td>45</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
</tr>
<tr>
<td>55</td>
<td></td>
</tr>
</tbody>
</table>

Correction coefficient \( (K) \) values for rated thermal current \( (I_{th}) \)
(ii) Equipment in enclosures

The above correction equation can be extended to include equipment in enclosures where the fluid temperature in the enclosure (the ambient for the equipment) is higher than the actual ambient outside the enclosure. If $I_{th}$ is the rating at an ambient of $40^\circ C$, then, when the ambient fluid temperature in the enclosure is $T_e$, the new rating is $I'_{th}$, given by the following equation. [$T_n$ is the maximum permissible temperature].

$$I'_{th} = KI_{th} = \left(\frac{T_n - T_e}{T_n - 40}\right)^{0.6} \times I_{th}$$

For example if $T_n = 75^\circ C$, $T_e = 45^\circ C$, then the de-rating factor is $K = 0.66$. The tables below show the typical increase of ambient temperature in some general enclosure situations.

<table>
<thead>
<tr>
<th>Type of enclosure</th>
<th>Number of transformers installed</th>
<th>250</th>
<th>500</th>
<th>750</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Underground vaults with natural ventilation</td>
<td>1</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>14</td>
<td>17</td>
<td>19</td>
<td>22</td>
</tr>
<tr>
<td>Basements and buildings with poor natural ventilation</td>
<td>1</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>10</td>
<td>13</td>
<td>15</td>
<td>17</td>
</tr>
<tr>
<td>Buildings with good natural ventilation and underground vaults and basements with forced ventilation</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>10</td>
<td>13</td>
</tr>
<tr>
<td>Kiosks</td>
<td>1</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>—</td>
</tr>
</tbody>
</table>

Correction due to increase in ambient temperature due to enclosure.

[AS2374.7-1997 Power transformers, Pt.7: Loading guide]
The actual fluid temperature in the enclosure can, in turn, be determined, by another correction formula, from the actual ambient temperature, using the filling factor of the enclosure.

\[ T_e = T_a + f \Delta T \]

where:  
\( T_e \) is the fluid temperature  
\( T_a \) is the external (true) ambient temperature  
\( \Delta T = T_{\text{max}} - T_a \) (\( T_{\text{max}} \) is temperature of the hottest surface in the enclosure)  
\( f \) is a filling coefficient obtained from the ratio of the volume of the equipment in the enclosure to the total enclosure volume.

Typically, in a 415 volt switchboard, for example, \( f = 0.25 \).

Thus, if \( T_{\text{max}} = 100 \, ^\circ C \)

\[ T_e = T_a + 0.25 (100 - T_a) \]
\[ = 0.75 T_a + 25 \, ^\circ C \]
\[ = 40 \, ^\circ C \] \text{[for } T_a = 20 \, ^\circ C \text{].}

For a very full enclosure, the filling factor would be \( f \approx 0.5 \).

The following tables give some further details of typical filling factors and corresponding corrections for effective ambient temperature.
For a normal sized poorly ventilated cell having a degree of enclosure IP4X to AS 1939, containing combined fuse switches and contactors, the value of \( f \) could be in the range 0.4 to 0.5. The value of \( f \) may reach 0.8 when equipment, such as circuit-breakers, are contained in very tight IP4X (see AS 1939) enclosures.

### TABLE 2.2.2

**Nominal Values of Correction Coefficient \( (K') \) for Unit in Enclosures with a Filling Coefficient \( f = 0.25 \) for Correcting the Rated Thermal Current \( (I_a) \)**

<table>
<thead>
<tr>
<th>Ambient Temperature ( (T_a) ) (^\circ)C</th>
<th>Correction coefficient ( (K') )</th>
<th>Maximum permissible temperature ( (T_m) ), (^\circ)C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>55</td>
<td>65</td>
</tr>
<tr>
<td>0</td>
<td>1.52</td>
<td>1.33</td>
</tr>
<tr>
<td>10</td>
<td>1.28</td>
<td>1.17</td>
</tr>
<tr>
<td>20</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>30</td>
<td>0.66</td>
<td>0.81</td>
</tr>
<tr>
<td>40</td>
<td>—</td>
<td>0.58</td>
</tr>
<tr>
<td>45</td>
<td>—</td>
<td>0.44</td>
</tr>
<tr>
<td>50</td>
<td>—</td>
<td>0.25</td>
</tr>
<tr>
<td>55</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

### TABLE 2.2.3

**Nominal Values of Correction Coefficient \( (K') \) for Units in Enclosures with a Filling Coefficient \( f = 0.5 \) for Correcting the Rated Thermal Current \( (I_a) \)**

<table>
<thead>
<tr>
<th>Ambient Temperature ( (T_a) ) (^\circ)C</th>
<th>Correction coefficient ( (K') )</th>
<th>Maximum permissible temperature ( (T_m) ), (^\circ)C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>55</td>
<td>65</td>
</tr>
<tr>
<td>0</td>
<td>0.52</td>
<td>0.74</td>
</tr>
<tr>
<td>10</td>
<td>—</td>
<td>0.58</td>
</tr>
<tr>
<td>20</td>
<td>—</td>
<td>0.44</td>
</tr>
<tr>
<td>30</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>40</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>45</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>50</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>55</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

The lowest value of \( T_a \) is to be used for the unit considered excluding minor components with low values of maximum permissible temperatures (e.g. push-buttons and accessible parts which can be touched) for which special precautions can be taken.

**NOTE:** The correction coefficients \( (K) \) and \( (K') \) are not applied to units for which the rated current is known to give \( T_a \) for the values of \( T_a \) and \( T_2 \) considered.

Where the unit contains components whose maximum permissible temperature is not attained for \( T_a = 40^\circ C \) and if the average ambient temperature exceeds \( 20^\circ C \) for long periods, it is possible to increase the maximum permissible temperature rises without the ageing of the components concerned being appreciably increased.
The effect of temperature rise of connecting conductors to electrical equipment

In addition to the rating of the main item of bulk equipment, it is often necessary to calculate the temperature rise of the conductors leading to the equipment. This is necessary because they may often represent a heat source or a heat sink for heat generated in the equipment and the resulting thermal conduction to or from the equipment will affect the equipment temperature rise. Thus, it is useful to elaborate on the means of calculating the temperature rise of such conductors. The processes are very similar to those used for overhead line conductors, although the environment of the conductors in this case may be more complex.

Heat dissipation in such cases is only by convection and radiation. Heat generation is by Ohmic heating and (on some occasions) solar absorption.

Power loss generation in the conductor:

(i) Ohmic heating

\[ H_{\text{ohmic}} = I^2 R_0 \left[ 1 + \alpha \left( T_e + \Delta T_s \right) \right] \text{ watts per metre} \]

- \( T_e \) is the fluid environment temperature (°C)
- \( \Delta T_s \) is the temperature rise of the conductor above \( T_e \) (°C)
$R_0$ is the electrical resistance per metre of the conductor at some specified temperature (usually 20°C) (ohms)

(ii) Solar absorption

\[ H_{\text{solar}} = a\Phi A_p \text{ watts per metre} \]

\(a\) is the radiation absorption coefficient of the surface

\(\Phi\) is the solar radiation flux density (W/m\(^2\)), its maximum value at the equator is 1100 W/m\(^2\)

\(A_p\) is the projected area of the conductor to the solar flux (m\(^2\)/m)

Heat dissipation from the conductor:

(i) Convection:

\[ H_{\text{conv}} = \left[ BLk/D \right] Nu\Delta T_s \]

\(B\) is the perimeter of the conductor surface (m)

\(L\) is the conductor length (m)

\(k\) is the thermal conductivity of the fluid

\(D\) is the conductor diameter or height (m)

\(Nu\) is the Nusselt number of the fluid

\(\Delta T_s\) is the temperature rise of the conductor above fluid temperature
The calculation thus requires calculation of the Nusselt number $Nu$. This in turn depends on the Prandtl ($Pr$) and Grashof ($Gr$) numbers of the fluid for natural convection and the Reynolds number for forced convection.

For example the following expression is often used for $Nu$ in natural convection:

$$
Nu = 0.8 \left( Gr \cdot Pr \right)^{0.05} + 0.35 \left( Gr \cdot Pr \right)^{0.27}
$$

where $Gr \cdot Pr = \left[ \rho^2 \beta c D^3 g \Delta T_s \right]/k \mu_d$

- $\rho$ is the fluid density
- $\beta$ is the coefficient of thermal expansion
- $c$ is the specific heat
- $g$ is the gravitational acceleration
- $\mu_d$ is the dynamic viscosity of the fluid
- $k$ is the thermal conductivity of the fluid

For forced convection, the Reynolds number ($Re$) rather than the $Gr$ and $Pr$ numbers must be used. For example, with applied fluid velocity $V$ m/s over a cylinder of diameter $D$:

$$
Nu = 0.65 Re^{0.2} + 0.23 Re^{0.61}
$$

$$
Re = \rho V D / \mu_d
$$

(ii) Radiation dissipation
Use the Stefan-Boltzmann Law

\[ H_r = BL\sigma\varepsilon\left[ (T_e + \Delta T_s + 273)^4 - (T_a + 273)^4 \right] \text{ watts} \]

\( \sigma \) is Stefan-Boltzmann constant
\( \varepsilon \) is emissivity of conductor surface, e.g. approx 0.05 for bright copper, 0.5 – 1 for oxidised Cu.

Thus, from the above balance of heat generation and heat dissipation, we get for the temperature rise \( \Delta T_s \):

\[
\Delta T_s = \frac{I^2R_0\left[ 1 + \alpha(T_e + \Delta T_s) \right] + a\Phi A_p}{BL\left[ \sigma\varepsilon\left( \frac{(T_e + \Delta T_s + 273)^4 - (T_a + 273)^4}{\Delta T_s} \right) + \frac{k\nu}{D} \right]}
\]

As can be seen, \( \Delta T_s \) can occur on both sides of the above equation. Thus the solution may require iteration. However only a few steps are usually required for convergence.

Once \( \Delta T_s \) is determined, it can be compared to the equipment temperature rise and the level of heat transferred to or from the equipment by thermal conduction along the conductor can then be calculated and the resulting temperature of the equipment determined.
1.3.2 **Short Circuit Rating**

When short circuits occur the electrical protection should operate within a very short time, typically a maximum of 1 to 3 seconds. During this period of short circuit current flow, the heating is adiabatic: there is essentially no heat loss to the ambient because of the short duration. All the heat remains within the equipment and causes its temperature to rise. The heat balance equation becomes:

$$ Pdt = mcd \theta $$

For short circuit heating, $P = I^2 R$

Thus:

$$ I^2 R dt = mcd \theta $$

and then

$$ \theta(t) - \theta_i = \left[ \frac{I^2 R}{mc} \right] t = \left[ \frac{R}{mc} \right] I^2 t $$

where $\theta_i$ is the initial temperature.

$I^2 t$ is the fault energy “let-through”. It is a very important quantity for any fault. It has units of joules per ohm. $RI^2 t$ is thus the total energy deposited in the resistance $R$ by the fault current $I$.

For:

$$ R = \frac{\rho L}{A} \quad \text{and} \quad m = \delta AL $$

$$ [\delta = \text{density}, L = \text{length}, A = \text{cross-section area}] $$
then: 

\[ \theta(t) - \theta_i = \left[ \frac{\rho}{\delta c} \right] \left[ \frac{I^2 t}{A^2} \right] \]

Thus the temperature rise in this simple analysis is then linear with the \( I^2 t \) for the fault.

However, because the resistance \( R \) is temperature dependent, \( I^2 R \) is not constant during the fault: we must use a temperature-dependent \( R \), given by the equation:

then: 

\[ R(\theta) = R_0 \left[ 1 + \alpha (\theta - \theta_0) \right] \]

we then get: 

\[ \frac{d\theta}{dt} = \frac{I^2 R_0}{mc} \left[ 1 + \alpha (\theta - \theta_0) \right] \]

\[ = \left( \frac{\rho_0}{\delta c} \right) \frac{I^2}{A^2} \left[ 1 + \alpha (\theta - \theta_0) \right] \]

This is a more complex differential equation to solve: the end result is that the temperature rise of the resistance on short circuit is faster than linear, as shown below.
The solution of the non-linear d.e. is:

$$\theta(t) = \frac{1}{\alpha} \left[ e^{k^2 j^2 t} - 1 \right] + (\theta_i - \theta_0) e^{k^2 j^2 t} + \theta_0$$

where:

$$k = \frac{\rho_0 \alpha}{\delta c} \quad \text{(a constant of the material)}$$

$$j = \frac{I}{A} \quad \text{(the fault current density)}$$

$$\theta_i \text{ is the initial temperature at fault onset, i.e. } \theta_i = \theta(0)$$

$$\theta_0 \text{ is the ambient temperature}$$

The more usual form for the above relationship is the expression:
\[
\frac{I^2 t_s}{A^2} = \frac{1}{k} \log_e \left( \frac{1 + \alpha \left( \theta(t_s) - \theta_0 \right)}{1 + \alpha \left( \theta(0) - \theta_0 \right)} \right)
\]

The curves on the following page show plots of the above relationship for the metals copper, aluminium, steel and lead.

From the above equations we can see some general features of the temperature response of equipment when subject to short circuit heating.

(i) For a given \( A \), \( \theta(t_s) - \theta_0 = \text{function} \ (I^2 t) \). Can match to the fuse \( (I^2 t) \) to limit \( \theta(t_s) \).

(ii) For a specified \( t_s \) (the protection operating time), a maximum current density \( j = I/A \) should be specified corresponding to an upper limit of \( \theta(t_s) \).
Note: vertical axis unit is \( \left( \frac{A}{m^2} \right) \sqrt{s} \), multiplier is \( 10^6 \).
Some typical maximum levels of temperature for short circuit conditions are:

**Overhead lines** [the limit is due to potential annealing effects]

- Copper: 200°C
- Aluminium: 180°C
- Steel: 200°C

**Underground cables** [the limit is determined by potential damage to insulation]

- Copper: 140°C
- Aluminium: 120°C

**Example**

The maximum permissible temperature for a 200 mm² copper cable is 120°C. If the initial temperature at fault onset is 70°C, determine, for fault clearance times of (a) 3 seconds and (b) 1 second:

(i) the maximum allowable fault current density and the fault current
(ii) the $I^2t$ values for the two fault clearance times.

We have: $\theta_m = 120 \, ^\circ\text{C}$ and $\theta_i = 70 \, ^\circ\text{C}$
The graphs give: \( \frac{I\sqrt{t}}{A} = 90 \, A\sqrt{s/mm^2} \Rightarrow \frac{I}{A} = \frac{90}{\sqrt{t}} \)

For: \( t_s = 1 \, s \) \( j = \frac{I}{A} = 90 \, A/mm^2 \)

\( t_s = 3 \, s \) \( j = \frac{I}{A} = 52 \, A/mm^2 \)

Thus: \( I = 18 \, kA \) (1 sec)

\( I = 10.4 \, kA \) (3 sec)

or: \( I^2 t = 90^2 \times A^2 = 8.1 \times 10^3 \, A^2 \)

For \( A = 200 \, mm^2 \)

\( I^2 t = \left(8.1 \times 10^3\right) \times \left(4 \times 10^4\right) = 3.24 \times 10^8 \, A^2 \cdot s \)

c.f. fuse \( I^2 t \).
1.3.3 Cyclic and Short Term ratings

If the thermal time constant of the equipment is high, it is possible to operate the equipment at higher than rated load for short periods without exceeding the maximum temperature. This is the basis for cyclic rating and short-term ratings of equipment. Both of these ratings are higher than the steady state thermal rating of the equipment.

(a) Short term rating

The general requirement is the specification of a loading level that can be sustained for a specific time: for example, one hour, without exceeding the maximum allowable temperature.

\[
\Delta \theta_{\text{st}} = 1.9 \Delta \theta_m \\
\Delta \theta_{\text{st}} = 1.5 \Delta \theta_m \\
0.63 \times \Delta \theta_m
\]

\(\Delta \theta\) vs. Time

\(\Delta \theta\) is the temperature difference due to the short-term loading. The steady-state rating is indicated by the flat line at the top of the graph. Short-term ratings are indicated by the curves that show how temperature changes over time.

\(l = \tau\) (time constant)

\(0.75\tau\) and \(1.1\tau\) are points on the graph that indicate the time points at which the short-term temperatures reach specific values relative to the steady-state rating.
\[ \Delta \theta_{ss} \propto P \]

Hence \[ \Delta \theta_{ss} \propto I^2 \]

Thus:

\[
\frac{I_{st}}{I_{th}} = \sqrt{\frac{\Delta \theta_{st}}{\Delta \theta_m}} \quad \Rightarrow \quad I_{st} = I_{th} \sqrt{\frac{\Delta \theta_{st}}{\Delta \theta_m}}
\]

This equation must be used to determine the allowable time duration of loading.

Alternatively, we can specify the time duration and then use the time constant \( \tau \) and the allowable \( \Delta \theta_m \) to determine the allowable \( I_{st} \).

**Example:**

(1) If \( \Delta \theta_{st} = 1.5 \Delta \theta_m \)

then if \( P \propto I^2 \), \( I_{st} = \sqrt{1.5} I_{th} = 1.23 \) p.u.

With \( I_{st} \), \( \Delta \theta(t) = 1.5 \Delta \theta_m \left( 1 - e^{-t/\tau} \right) \)

But: \( \Delta \theta = \Delta \theta_m \) at \( t = t_0 \)

Hence:

\[
\frac{1}{1.5} = 1 - e^{-t_0/\tau} \quad \Rightarrow \quad t_0 = 1.1 \tau
\]

Thus, \( I_{st} \) is allowed only for \( t = 1.1 \tau \) hours.
(2) If we specify a load time of \( t_1 = 0.75\tau \)

then: \( \Delta \theta_m = \Delta \theta_{st} \left( 1 - e^{-0.75} \right) \)

Hence: \( \Delta \theta_{st} = 1.895\Delta \theta_m \)

And if \( \Delta \theta_{ss} \propto I^2 \), this gives:

\[
I_{st} = \sqrt{1.895I_{th}} = 1.38 \text{ pu for } 0.75 \tau \text{ hours.}
\]

(b) Cyclic rating

Cyclic rating is essentially a series of short duration periods of loading. The aim as before is to determine a loading level that will not have the temperature exceed the permissible level for the insulation.
From the above diagram it can be seen that the final “steady state” situation is a temperature oscillation between two elevated temperature rise levels, $\Delta \theta_{\text{max}}$ and $\Delta \theta_{\text{min}}$.

We define a cyclic rating factor:

$$K = \sqrt{\frac{\Delta \theta_{cr}}{\Delta \theta_{\text{max}}}}$$

This rating factor is such as to make it possible to increase the rating to

$$I_{cr} = K \times I_{th}$$

for cyclic loading without exceeding the temperature rise limit $\Delta \theta_{\text{limit}}$ specified for the insulation.

This is commonly used for both cables and transformers which both have high thermal time constants and which are also commonly subjected to a daily loading cycle.

Unfortunately, it is not generally the case that the loading is so well specified as above. It may vary considerably. However, general experience has determined the most likely variation forms and these are used in Standards to calculate cyclic loading factors for transformers, for example.

The following shows a cyclic rating table for oil filled transformers.
2.6.1.2 **Load cycle with one peak.** A typical load cycle with one peak is shown in Fig. 2.3.

In this case the value of $h$ should be selected on an area basis as indicated.

For the off-peak portion of the load cycle, the value of $k_1$ is selected to correspond to the average off-peak load.

---

**Fig. 2.3. LOAD CYCLE WITH ONE PEAK**
The following gives the detailed derivation of the cyclic rating factors for well-defined regular and periodic cyclic loadings. The importance of the thermal time constant in this cyclic loading factor is evident.
Assuming same time constant $\tau$ for both heating and cooling, we have:

\[
\Delta \theta_2 - \Delta \theta_1 = \left( \Delta \theta_m - \Delta \theta_1 \right) \left( 1 - e^{-h/\tau} \right) \quad \text{[heating]}
\]

\[
\Delta \theta_1 = \Delta \theta_2 e^{-T_1/\tau} \quad \text{[cooling]}
\]

Thus:

\[
\Delta \theta_2 - \Delta \theta_2 e^{-T_1/\tau} = \left( \Delta \theta_m - \Delta \theta_2 e^{-T_1/\tau} \right) \left( 1 - e^{-h/\tau} \right)
\]

or:

\[
\Delta \theta_2 - \Delta \theta_2 e^{-T_1/T} = \Delta \theta_m - \Delta \theta_m e^{-t_1/T} \quad (T_2 = t_1 + T_1)
\]
\[ \Delta \theta_2 = \Delta \theta_m \left( \frac{1 - e^{-t_1/\tau}}{1 - e^{-T_2/\tau}} \right) \]

Cyclic rating factor is:

\[ K = \sqrt{\frac{\Delta \theta_m}{\Delta \theta_2}} \quad \text{or} \quad \sqrt{\frac{\theta_m - \theta_0}{\theta_2 - \theta_0}} = \sqrt{\frac{1 - e^{-T_2/\tau}}{1 - e^{-t_1/\tau}}} \]

The cyclic rating factor is the multiplying factor on full-load rating that is allowed for time \( t_1 \) and cycle period \( T_2 \). At a cyclic load \( KI_{th} \), the temperature will oscillate between \( \theta_2 \) and \( \theta_1 \).

An alternative derivation using the actual temperatures, rather than rises is:

\[ \theta_2 - \theta_1 = (\theta_m - \theta_1)(1 - e^{-t_1/\tau}) \quad \text{(Eq.1)} \]

\[ \theta_1 - \theta_0 = (\theta_2 - \theta_0)e^{-T_1/\tau} \quad \text{(Eq.2)} \]

Use Eq.2 for \( \theta_1 \) and substitute into Eq.1:

\[ \theta_2 - (\theta_2 - \theta_0)e^{-T_1/\tau} - \theta_0 = (\theta_m - [\theta_2 - \theta_0]e^{-T_1/\tau} - \theta_0)(1 - e^{-t_1/\tau}) \]

or, we can write equations (1) and (2) as:

\[ (\theta_2 - \theta_0) - (\theta_1 - \theta_0) = \left[ (\theta_m - \theta_0) - (\theta_1 - \theta_0) \right](1 - e^{-t_1/\tau}) \]

\[ \theta_1 - \theta_0 = (\theta_2 - \theta_0)e^{-T_1/\tau} \]
Then in the same form as previously, we have:

\[ \theta_2 - \theta_0 = (\theta_m - \theta_0) \left( \frac{1 - e^{-t_1/\tau}}{1 - e^{-T_2/\tau}} \right) \]

and the cyclic rating factor is:

\[ \sqrt{\frac{\theta_m - \theta_0}{\theta_2 - \theta_0}} = \sqrt{\frac{1 - e^{-T_2/T}}{1 - e^{-t_1/T}}} \]

as before.
**APPENDIX**

**Problem:** find solution to

\[
\frac{dx}{dt} + \frac{x(t)}{\tau} = K \quad \text{Eq.1}
\]

This is a first-order differential equation. The parameter \( \tau \) is called the **time constant**. We solve this equation by separating the variables and then integrating. Rearrange:

\[
\frac{dx}{dt} = \frac{K\tau - x}{\tau}
\]

\[
\frac{dx}{x-K\tau} = -\frac{dt}{\tau}
\]

\[
\int \frac{dx}{x-K\tau} = -\frac{1}{\tau} \int dt + D
\]

where D is a constant of integration. Integrate:

\[
\ln(x-K\tau) = -\frac{t}{\tau} + D
\]

\[
x-K\tau = \exp\left(-\frac{t}{\tau} + D\right)
\]

\[
x(t) = K\tau + e^D e^{-t/\tau}
\]

At \( t=0 \):

\[
x(0) = K\tau + e^D \quad \Rightarrow \quad e^D = x(0) - K\tau
\]

Hence:

\[
x(t) = K\tau + (x(0) - K\tau)e^{-t/\tau} \quad \text{Eq.2}
\]
Now consider:

\[ Pdt = mc d\theta + Ah (\theta - \theta_o) dt \]

Rearrange:

\[ \frac{d}{dt} \theta(t) + \frac{\theta(t)}{\tau} = K \]

where:

\[ \tau \equiv \frac{mc}{Ah} \quad \text{and} \quad K \equiv \frac{1}{\tau} \left( \frac{P}{Ah} + \theta_o \right) \quad \text{Eq.3} \]

Thus from Eq.2 and initial condition: \( \theta(0) = \theta_o \):

\[ \theta(t) = K \tau + [\theta_o - K \tau] e^{-t/\tau} \quad \text{Eq.4} \]

As \( t \to \infty \):

\[ \theta(\infty) = K \tau = \frac{P}{Ah} + \theta_o \equiv \theta_{ss} \]

\[ \theta_{ss} - \theta_o \equiv \Delta \theta_{ss} = \frac{P}{Ah} \]

Thus,

\[ K \tau = \Delta \theta_{ss} + \theta_o \quad \text{Eq.5} \]

Hence Eq.4 becomes:

\[ \theta(t) = \Delta \theta_{ss} + \theta_o + \Delta \theta_{ss} e^{-t/\tau} \]

i.e.

\[ \theta(t) - \theta_o = \Delta \theta_{ss} \left[ 1 - e^{-t/\tau} \right] \]
If the initial condition is $\theta(0) = \theta_1$ then Eq. 2 becomes:

$$\theta(t) = K \tau + [\theta_1 - K \tau] e^{-t/\tau}$$

$$= \Delta \theta_{ss} + \theta_o + \left[ \theta_1 - (\Delta \theta_{ss} + \theta_o) \right] e^{-t/\tau}$$

$$= (\theta_{ss} - \theta_o) + \theta_o + \left[ \theta_1 - (\theta_{ss} - \theta_o + \theta_o) \right] e^{-t/\tau}$$

$$= \theta_{ss} + [\theta_1 - \theta_{ss}] e^{-t/\tau}$$

Hence: $\theta(t) - \theta_1 = \theta_{ss} - \theta_1 + (\theta_1 - \theta_{ss}) e^{-t/\tau}$

i.e.

$$\theta(t) - \theta_1 = (\theta_{ss} - \theta_1) \left[ 1 - e^{-t/\tau} \right]$$