Discrete Non-Minimal State Space by Example

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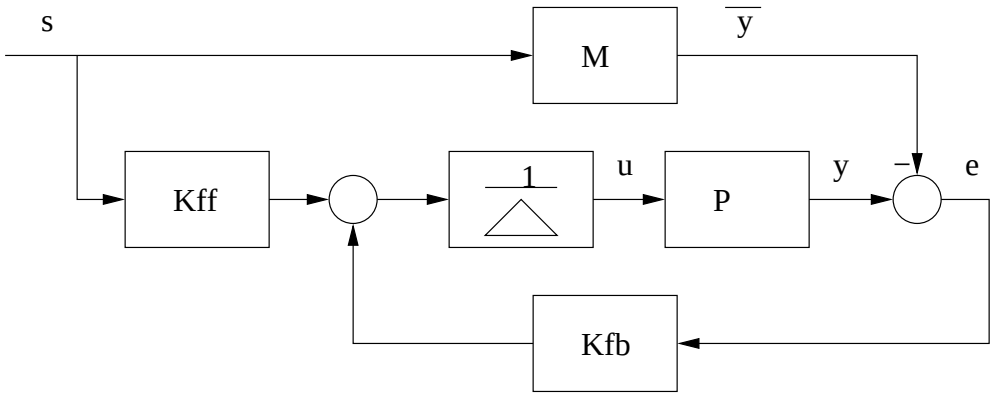
Abstract

Notes are provided on the web describing how to form discrete non-minimal state-space models. Here, a simple example is provided showing how to form the NMSS model. An example is provided at the end as a class exercise.

Real Time Computing & Control

What are we trying to achieve?

Ultimately, we want to design a controller that looks like this:



- s is the setpoint
- $\bar{y}$ is the setpoint filtered by M to be the sahape we want to follow
- P is the process, with input u and output y
- K_{ff} and K_{fb} are the controller transfer functions (feedforward and feedback)
- $\frac{1}{\Delta}$ is an integrator

We want to put the overall model into a state space form that will allow us to design this controller.

Example System

Suppose

$$e = Pu - Ms = \frac{q^{-1}}{1 - 0.8q^{-1}}u - \frac{0.5q^{-1}}{1 - 0.5q^{-1}}s$$

Multpying both sides by $\Delta = (1 - q^{-1})$

$$\begin{aligned}(1 - 2.3q^{-1} + 1.7q^{-2} - 0.4q^{-3})e &= (q^{-1} - 0.5q^{-2})\Delta u + \\ &\quad (-0.5q^{-1} + 0.4q^{-2})\Delta s\end{aligned}$$

We can write this as a state space equation

$$\begin{aligned}x(t) &= \mathcal{A}x(t-1) + \mathcal{B}\Delta u(t-1) + \mathcal{E}\Delta s(t) \\ e(t) &= \mathcal{C}x(t)\end{aligned}$$

NMSS Structure

State Vector

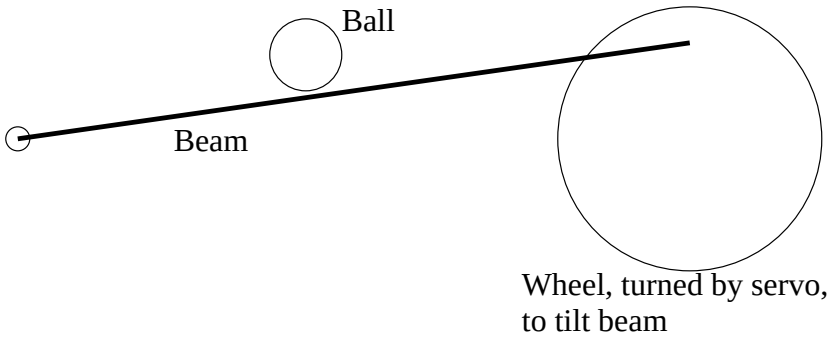
$$x(t) = \begin{bmatrix} e(t) & e(t-1) & e(t-2) & \dots \\ \dots & \Delta u(t-1) & \Delta s(t) & \Delta s(t-1) &]^\top \end{bmatrix}$$

$\mathcal{A}, \mathcal{B}, \mathcal{C}$

$$\left[\begin{array}{cccccc|c} 2.3 & -1.7 & 0.4 & -0.5 & -0.5 & 0.4 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 & - \end{array}\right]$$

Be sure you know how to get these. Write out the NMSS equations in full. $\mathcal{E}$ is not important for control, but work it out.

Exercise - Ball and Beam



Variables y: horizontal position of ball on beam; u: voltage to servomotor

Process Transfer Function

$$\frac{y}{u} = \frac{4.16}{s^2}$$

Set Point Filter

$$M = 0.1q^{-1} + 0.2q^{-2} + 0.4q^{-3} + 0.2q^{-4} + 0.1q^{-5}$$

What to do? Write the NMSS equations for the complete system.