Analogue signal transmission and reception:

Analogue signals (e.g., speech, music etc.) can be modulated and transmitted directly or can be converted to digital data and transmitted using digital modulation techniques.

In spite of the general trend toward digital transmission of analogue signals, there is still today a significant amount of analogue signal transmission especially in audio and video broadcast.

In this section we treat the transmission of analogue signals by carrier modulation.

Modulation:

The analogue signal to be transmitted is denoted by \( m(t) \), which is assumed to be a low-pass signal of bandwidth \( B \).

In other words:

\[
M(f) = 0 \text{ for } |f| > B
\]

The message signal \( m(t) \) is transmitted through the communication channel by impressing it on a carrier signal of the form.

\[
c(t) = A_c \cos(\omega_c t) \quad \omega_c = 2\pi f_c
\]

Suppose the modulating signal (message) \( m(t) \) is a sinusoid, then \( m(t) \) is given by:

\[
m(t) = A_m \cos(\omega_m t) \quad \omega_m = 2\pi f_m
\]

where \( \omega_c \gg \omega_m \) or \( f_c \gg f_m \)

Sometimes the carrier signal \( c(t) \) is written as
\[ c(t) = A_c \cos(\omega_c t + \Phi_c) \], where \( \Phi_c \) is the carrier phase.

We say that the message signal \( m(t) \) modulates the carrier signal \( c(t) \) in **amplitude**, **frequency** or **phase**, if after modulation the amplitude, frequency, or phase, of the signal become functions of the message signal.

In effect, modulation converts the message \( m(t) \) from low pass to band pass, in the neighborhood of the centre frequency \( f_c \).

\[
\begin{array}{c}
\text{Low Pass} \\
\text{Band Pass}
\end{array}
\]

The three modulation methods for the transmission and reception of analogue signals are:

- **Amplitude Modulation (AM)**
- **Frequency Modulation (FM)**
- **Carrier-Phase Modulation (PM)**

**Amplitude Modulation:**

In amplitude modulation, the message signal \( m(t) \) is impressed on the amplitude of the carrier signal \( c(t) \). There are several different ways of amplitude modulating the carrier signal by \( m(t) \), each of which results in **different spectral characteristics** for the transmitted signal.
1. Conventional double-sideband AM:

Consider a sinusoidal carrier wave \( c(t) \) defined by

\[
c(t) = A_c \cos(\omega_c t + \Phi_c)
\] (1).

For convenience, we assume that the phase of the carrier wave is zero

\[
\therefore \ c(t) = A_c \cos(\omega_c t)
\] (2).

Let \( m(t) \) denote a message signal of interest. Amplitude modulation (AM) is defined as a process in which the amplitude of the carrier is varied proportionately to a message signal \( m(t) \), as shown by

\[
s(t) = A_c [1 + k_a m(t)] \cos(\omega_c t)
\] (3).

Where, \( k_a \) is a constant called, the amplitude sensitivity factor of the modulator.

Note: The frequency \( \omega_c \) of the carrier is maintained constant; but its amplitude is varied according to the message signal \( m(t) \). [See equation (3) above]
\[ s(t) = A_c [1 + k_a m(t)] \cos(\omega_c) \]

Let \[ A(t) = A_c [1 + k_a m(t)] \quad (4). \]

Two cases arise depending on the magnitude of \( k_a m(t) \), compared to unity.

1. \(|k_a m(t)| \leq 1\) for all \( t \)

Under this condition, the term \( 1 + k_a m(t) \) is always nonnegative.

2. \(|k_a m(t)| > 1\) for some \( t \)

Under this condition, we must use \( A(t) = A_c [1 + k_a m(t)] \) for evaluating the envelope of the AM wave.
A waveform illustrating the AM signal, as seen on an oscilloscope is shown in figure 2.

Where $A_{\text{max}}$ and $A_{\text{min}}$ denote the maximum and minimum values of the envelope of the modulated wave and $A_c$ is the level of the AM envelope in the absence of modulation. [i.e. when $m(t) = 0$ i.e. unmodulated carrier wave.]

Consider a modulating wave $m(t)$ that consists of a single frequency component.

$$m(t) = A_m \cos(\omega_m t) \quad (5).$$

Amplitude of the modulating signal
The AM wave is described by [see fig. 2]

\[ s(t) = A_e[1 + k_a A_m \cos(\omega_m t)]\cos\omega_c t \quad (6). \]

Let \( \mu = k_a A_m \) (\( \mu \) - modulation index)

\[ s(t) = A_e[1 + \mu \cos(\omega_m t)]\cos\omega_c t \quad (7). \]

Using fig. 2 & equation (7) we get

\[ A_{\text{max}} = A_e[1 + \mu] \quad (8). \]
\[ A_{\text{min}} = A_e[1 - \mu] \quad (9). \]

\[ \frac{A_{\text{max}}}{A_{\text{min}}} = \frac{A_e[1 + \mu]}{A_e[1 - \mu]} \quad (10). \]

Solving for \( \mu \), using (10).

\[ \mu = \frac{A_{\text{max}} - A_{\text{min}}}{A_{\text{max}} + A_{\text{min}}} \quad (11). \]

\( \mu \), is known as modulation index.

\textbf{Case 1:} \quad A_{\text{max}} = A_{\text{min}} \quad \mu = 0 \text{ (no modulation)}

\textbf{Case 2:} \quad A_{\text{max}} = 2A_e, A_{\text{min}} = 0 \quad \mu = 1 \text{ (100% modulation)}

\textbf{Case 3:} \quad A_{\text{max}} > A_{\text{min}} \quad \mu < 1 \text{ (under modulation <100%)}

\textbf{Case 4:} \quad A_{\text{min}} < 0 \quad \mu > 1 \text{ (over modulation>100%), causes phase reversals & envelope distortion}
AM Modulated Waves:

Comparing the waveforms of the above three waves with that of message signal $m(t)$, we draw an important conclusion. The envelope of the AM wave has a waveform that bears a one-to-one correspondence with that of the message signal if and only if the percentage modulation

- Case 3: Modulation index $\mu < 1$ e.g. $\mu = 0.66$ (66% modulation)
- Case 2: Modulation index $\mu = 1$
- Case 4: Modulation index $\mu > 1$ e.g. $\mu = 1.66$ (166% modulation)
is less than or equal to 100%. If $\mu > 1$, this correspondence is destroyed and the waveform is said to suffer from envelope distortion and the wave itself is said to be **over-modulated**.

Consider equation (7) (page…)

$$s(t) = A_c [1 + \mu \cos \omega_c t] \cos \omega t$$  \hspace{1cm} (12).

**Note:**

\[ f_c \gg f_m \]
\[ \mu = k_0 A_m \]

Amplitude of modulating signal

\[
s(t) = A_c \cos \omega_c t + A_c \mu \cos \omega_m t \cos \omega_c t \\
= A_c \cos \omega_c t + \mu A_c \left[ \frac{\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t}{2} \right] \Rightarrow \\
s(t) = A_c \cos \omega_c t + \frac{\mu A_c}{2} \cos(\omega_c + \omega_m)t + \frac{\mu A_c}{2} \cos(\omega_c - \omega_m)t
\]

(12).

**Note: Frequency Domain Description**

\[
FT(\cos \omega_c t) = FT\left\{ \frac{e^{j \omega_c t} + e^{-j \omega_c t}}{2} \right\} = \frac{1}{2} FT\{e^{j \omega_c t}\} + \frac{1}{2} FT\{e^{-j \omega_c t}\}
\]

\[
FT(\cos \omega_c t) = \pi \delta(\omega - \omega_c) + \pi \delta(\omega + \omega_c)
\]

*Fourier Transform (FT) representation of the cosine function*
The Fourier transform of \( s(t) \) is therefore

\[
s(t) = A_c \cos \omega_c t + \frac{\mu A_c}{2} \cos(\omega_c + \omega_m) t + \frac{\mu A_c}{2} \cos(\omega_c - \omega_m) t
\]

The Fourier transform of \( s(t) \) is therefore

\[
s(\omega) = \pi A_c [\delta(\omega - \omega_c) + \delta(\omega + \omega_c)] + 1/2 \mu A_c [\delta(\omega - (\omega_c - \omega_m)) + \delta(\omega + (\omega_c + \omega_m))] + 1/2 \mu A_c [\delta(\omega - (\omega_c - \omega_m)) + \delta(\omega + (\omega_c - \omega_m))] \quad (13).
\]

Thus in ideal terms, the spectrum of a full AM wave, [for sinusoidal modulation], consists of impulse functions at \( \pm \omega_c, \omega_c \pm \omega_m, -\omega_c \pm \omega_m \) as shown in figure 3.
Let the message signal $m(t)$, be band-limited to the interval $-\omega_b \leq \omega \leq \omega_b$.

The shape of the spectrum shown in the above figure 4(b) is intended for the purpose of illustration only.
For perspective frequencies, the portion of the spectrum of the modulated wave lying above
the carrier frequency \( f_c \) is called the upper sideband, whereas, the symmetric portion
below \( f_c \) is called lower sideband.

For negative frequencies, the image of the upper sideband is represented by the portion of
the spectrum below \( -\omega_c \) and the image of the lower sideband by the portion above \( -f_c \).
The condition \( \omega_c \gg \omega_m \) ensures that the sidebands do not overlap.

For positive frequencies, the highest frequency component of the AM wave is \( f_c + f_b \) and
the lowest frequency component is \( f_c - f_b \).

The difference between these two frequencies defines the transmission bandwidth \( f_T \) for
an AM wave.

Transmission BW \( f_T = 2f_B \) (twice the message bandwidth)  \( (15) \).

Message Bandwidth = \( f_B - 0 = f_B \)
Power content of the AM wave

\[ s(t) = A_c (1 + \mu \cos \omega_m t) \cos \omega_c t = \]

\[ A_c \cos \omega_c t + \left( \frac{\mu A_c}{2} \right) \cos(\omega_c + \omega_m) t + \left( \frac{\mu A_c}{2} \right) \cos(\omega_c - \omega_m) t \] (16).

In practice, the AM wave \( s(t) \) is a voltage or current signal. In either case, the average power delivered to a \( 1 \Omega \) resistor by \( s(t) \) comprised of three components:

Carrier Power (\( P_c \)) = \( \left( \frac{A_c}{\sqrt{2}} \right)^2 = \frac{A_c^2}{2} \) (17).

Upper side frequency power (\( P_u \)) = \( \left( \frac{\mu A_c}{2\sqrt{2}} \right)^2 = \frac{\mu^2 A_c^2}{8} \).
Lower Side-frequency Power \( (P_L) = \left(\frac{1}{2} \mu \frac{A_c}{\sqrt{2}}\right)^2 = \frac{\mu^2 A_c^2}{8} \) \hspace{1cm} (18).

\[ \eta = \frac{\text{Total Sideband Power}}{\text{Total Modulated Wave Power}} = \frac{P_u + P_L}{P_c + P_u + P_L} = \frac{\frac{\mu^2 A_c^2}{8} + \frac{\mu^2 A_c}{8}}{\frac{A_c^2}{2} + \frac{\mu^2 A_c}{8} + \frac{\mu^2 A_c^2}{8}} = \frac{2\mu^2 A_c^2/8}{(4A_c^2 + 2\mu^2 A_c^2)/8} = \frac{\mu^2}{2 + \mu^2} \] \hspace{1cm} (19).

If \( \mu = 1 \), that is 100\% modulation is used, the total power in the two side-frequencies of the resulting AM wave is only \( \frac{1}{3} \) of the total power in the modulated wave.

\[ \dot{\cdot} \dot{\cdot} \text{ The efficiency (} \eta \text{) of the modulation process is defined as the ratio of the sideband power to the total power in the transmitted signal.} \]

\[ \dot{\cdot} \dot{\cdot} \text{ When } \mu = 1 \text{ (100\% modulation) } \eta = \frac{\mu^2}{2 + \mu^2} \times 100 = 33.3\% \] \hspace{1cm} (20).

Efficiency obviously declines rapidly as the modulation index \( \mu \) is reduced below unity.

Note that if we allow the modulation index to exceed one \( (\mu > 1) \), efficiency can exceed 50\%.

This figure shows the percentage of total power in both-side frequencies and in the carrier plotted against the \% modulation.
**Generation of AM wave**

Here we consider a sample circuit using the following equation:

\[ s(t) = A_c [1 + k_a m(t)] \cos \omega_c t \]  
(see equation 3 page 12)

We rewrite this equation in the equivalent form:

\[ s(t) = k_a [m(t) + B] A_c \cos \omega_c t \]  
(21).

The constant \( B = \frac{1}{k_a} \) represents a ‘bias’ that is added to the message signal \( m(t) \) before modulation.

Using equation (22), a scheme can be described for generating an AM wave (see figure 6).

The percentage modulation is controlled by adjusting the bias \( B \).

The constant \( k_a \) is a proportionality constant associated with the multiplier.

**Demodulation of AM wave:**

The so-called envelope detector provides a sample and yet effective device for the demodulation of a “narrowband” AM wave for which the percentage modulation is less than 100% \( (\mu < 1) \).

By ‘narrow band’ we mean that the carrier frequency is large compared with the message bandwidth.
Ideally, an envelope detector produces an output signal that follows the envelope of the input signal $s(t)$ exactly.

-Hence the name.

Figure 7 shows the circuit diagram of an envelope detector that consists of a diode and a resistor-capacitor filter.

The operation of the above circuit is as follows:

On the positive half-cycle of the input signal, the diode is forward biased and the capacitor $C$ charges up rapidly to the peak value of the input signal.
When the input signal falls below this value, the diode becomes reverse biased and the capacitor $C$ discharges slowly through the load resistor $R_L$. This discharging process continues until the next positive half cycle.
When the input signal becomes greater than the voltage across the capacitor, the diode conducts again and the process is repeated.
We assume that the diode is ideal, presenting zero impedance to current flow in the forward-biased region and infinite impedance in the reverse-biased region.
We further assume that the AM wave applied to the envelope detector is supplied by a voltage source of internal resistance $R_s$. The charging time constant $R_sC$ must be short compared with the carrier period $T_c = \frac{2\pi}{\omega_c}$, i.e. $R_sC \ll \frac{2\pi}{\omega_c}$.

The capacitor $C$ charges rapidly and thereby follows the applied voltage up to the positive peak when the diode is conducting (path A). On the other hand, the discharging time constant $R_LC$ must be long enough to ensure that the capacitor discharges slowly through the load resistor $R_L$ between positive peaks of the carrier wave (path C), but no so long (path D) that the capacitor will not discharge at the maximum rate of charge of the modulating wave.

That is:

**A**: Charging path

**B, C, D**: Discharging paths

**B**, $R_LC$ (time constant – too small)

**C**, $R_LC$ (time constant – reasonably correct)

**D**, $R_LC$ (time constant – too large)
\[
\frac{2\pi}{\omega_c} \ll R \frac{C}{C_m} \ll \frac{2\pi}{\omega_m}
\]
\[
\frac{1}{f_c} \ll R \frac{C}{C_m} \ll \frac{1}{f_m}
\]
\[
T_c \ll R \frac{C}{C_m} \ll T_m
\]

\(\omega_m\) is the message bandwidth.

The result is that the capacitor voltage or detector output is very nearly same as the envelope of the AM wave (see figure 7).

The detector output usually has a small ripple at the carrier frequency; this ripple is easily removed by low pass filtering.

**Exercise:**
You are given a nonlinear device, whose input-output relation is described by
\[
i_0 = a_1 v_i + a_2 v_i^2,\]
where \(a_1\) & \(a_2\) are constants, \(v_i\) is the input voltage, and \(i_0\) is the output current.
Let \(v_i(t) = A_c \cos \omega_c t + A_m \cos \omega_m t\)
\(\uparrow \text{carrier} \quad \uparrow \text{modulating signal}\)

(a) Determine the frequency content of \(i_0(t)\).

(b) Describe the specification of a filter that extracts the AM signal form \(i_0(t)\).

**Suggested Solutions:**
\[ i_0(t) = a_1 v_i(t) + a_2 [v_i(t)]^2 \]  \hspace{0.5cm} (A)

The input \( v_i(t) = A_c \cos(\omega_c t) + A_m \cos(\omega_m t) \)  \hspace{0.5cm} (B)

\[
\uparrow \quad \uparrow \quad \text{carrier wave} \quad \text{modulating wave}
\]

Substituting equation (B) in (A),

\[
i_0(t) = a_1[A_c \cos(\omega_c t) + A_m \cos(\omega_m t)] + a_2[A_c \cos(\omega_c t) + A_m \cos(\omega_m t)]^2 = a_1 A_c \cos(\omega_c t) + a_1 A_m \cos(\omega_m t) + a_2 A_c^2 \cos^2(\omega_c t) + 2 a_2 A_c A_m \cos(\omega_c t) \cos(\omega_m t) + a_2 A_m^2 \cos^2(\omega_m t)
\]

Using the trigonometric identity \( \cos^2 \theta = \frac{1}{2} (\cos 2\theta + 1) \), \( i_0(t) \) can be written as,

\[
\therefore i_0(t) = a_1 A_c \cos(\omega_c t) + 2 a_2 A_c A_m \cos(\omega_c t) \cos(\omega_m t) + a_1 A_m \cos(\omega_m t) + \frac{1}{2} a_2 A_m^2 \cos(2\omega_m t) + \frac{1}{2} a_2 A_c^2 \cos(2\omega_c t) + \frac{1}{2} a_2 A_m^2 + \frac{1}{2} a_2 A_c^2
\]

\[
\therefore i_0(t) = a_1 A_c \left( 1 + \frac{2 a_2 A_m}{a_1} \cos(\omega_m t) \right) \cos(\omega_c t) + \text{A set of undesirable components (i.e. DC, } \omega_m, 2\omega_m, 2\omega_c) \]

AM Component

Normally in AM \( \omega_c >> \omega_m \). The frequency content of \( i_0(t) \) may be represented as follows (showing only the positive frequency components).

\[ |I_0(\omega)| \]

\[ \omega_m \quad 2\omega_m \quad \omega_c - \omega_m \quad \omega_c \quad \omega_c + \omega_m \quad 2\omega_c \quad \omega \]

To extract these components we need to pass \( i_0(t) \) through a band pass filter.
Band-Pass filter specifications:

- Pass band of width $2\omega_m$, centred on the centre frequency $\omega_c$.
- Lower stop band must be below $\omega_c - \omega_m$ and thereby suppressing the dc component, $\omega_m & 2\omega_m$.
- Upper stop band should suppress $2\omega_c$.

Square-Law Modulator

Signal multiplication at high frequencies can be achieved by the square-law modulator.

$$v_0(t) = a_1 A_c (1 + \frac{2a_2 A_m}{a_1} \cos(\omega_m t)) \cos \omega_c t + \text{unwanted terms}$$

AM Wave

(see pages 26, 27)